Web Appendix to New Trade Models, New Welfare Implications (Not for Publication)*

Marc J. Melitz
Harvard University, NBER, and CEPR

Stephen J. Redding
Princeton University, NBER, and CEPR

August 13, 2014

1 Introduction

This web appendix contains the technical derivations of expressions for each section of the paper and additional supplementary material. In the interests of clarity, and to ensure that the web appendix is self-contained, we reproduce some material from the paper, but also include the intermediate steps for the derivation of expressions. Sections 2-7 of the web appendix closely follow the sections of the paper with the same name.

Section 8 provides a formal treatment of the revealed preference intuition discussed in the paper. The heterogeneous firm model has an additional adjustment margin that is absent in the homogeneous firm model (namely endogenous firm selection). With constant elasticity of substitution (CES) preferences, adjustment along this additional margin is efficient. As a result, if the degenerate productivity distribution in the homogeneous firm model is chosen so that the two models have the same welfare for an initial value of trade costs, the heterogeneous firm model has higher welfare for all other values of trade costs. As part of this analysis, we show that the social planner's problem can be reduced to a choice of the productivity cutoffs in an unconstrained maximization problem. In Section 9, we show that the social planner's problem also has an equivalent representation in terms of a constrained maximization problem.

In Section 10, we use this equivalent representation as a constrained maximization problem to consider the case of variable elasticity substitution (VES) preferences. In this case, the market equilibrium need not be efficient. Nonetheless, endogenous firm selection has the potential to generate higher welfare in the heterogeneous firm model than in the homogeneous firm model, as long as adjustment along this additional margin is similar in the market equilibrium and social optimum.

2 Heterogeneous and Homogeneous Firm Models

We compare the canonical heterogeneous firm model of Melitz (2003) to a homogeneous firm model that is a special case with a degenerate productivity distribution (as in Krugman 1980). We hold all other parameters (including the trading technology) constant across the two models.

*We are grateful to Harvard and Princeton Universities for research support and David Krisztian Nagy for research assistance. Responsibility for any results, opinions and errors is the authors' alone.

2.1 Closed Economy Heterogeneous Firm Model

The specification of preferences, production and entry is the same as Melitz (2003).[1] There is a continuum of firms that are heterogeneous in terms of their productivity $\varphi \in (0, \varphi_{\max})$, which is drawn from a fixed distribution $g(\varphi)$ after incurring a sunk entry cost of f_e units of labor. We allow the upper bound of the support of the productivity distribution to be either finite ($\varphi_{\max} < \infty$) or infinite ($\varphi_{\max} = \infty$). Production involves a fixed production cost and a constant marginal cost that depends on firm productivity, so that $l(\varphi) = f_d + q(\varphi)/\varphi$ units of labor are required to supply $q(\varphi)$ units of output. Consumers have constant elasticity of substitution (CES) preferences defined over the differentiated varieties supplied by firms, so that the equilibrium revenue for a firm with productivity φ is:

$$r(\varphi) = RP^{\sigma-1}p(\varphi)^{1-\sigma}, \tag{1}$$

where R is aggregate revenue; P is the aggregate CES price index; and $p(\varphi)$ is the price chosen by a firm with productivity φ. Profit maximization implies that equilibrium prices are a constant mark-up over marginal cost:

$$p(\varphi) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}. \tag{2}$$

Together the equilibrium revenue function (1) and pricing rule (2) imply that the relative revenues of firms depend only on their relative productivities:

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}, \tag{3}$$

and equilibrium profits are a constant fraction of revenue minus the fixed production cost:

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf_d.$$

Fixed production costs imply a productivity cutoff below which firms exit (φ_d^A) defined by the following zero-profit condition:

$$r_d(\varphi_d^A) = R\left(\frac{\sigma-1}{\sigma}P\varphi_d^A\right)^{\sigma-1} w^{1-\sigma} = \sigma w f_d, \tag{4}$$

where the superscript A denotes autarky.

The equilibrium value of this zero-profit productivity is uniquely determined by the free entry condition that requires that the probability of successful entry times average profits conditional on successful entry to equal the sunk entry cost:

$$\left[1 - G\left(\varphi_d^A\right)\right]\bar{\pi} = wf_e, \tag{5}$$

which using the relationship between the revenues of firms with different productivities (3) and the zero-profit condition (4) can be re-written as follows:

$$\begin{aligned}
\left[1 - G\left(\varphi_d^A\right)\right]\int_{\varphi_d^A}^{\varphi_{\max}}\left[\frac{r(\varphi)}{\sigma} - wf_d\right]\frac{\mathrm{d}G(\varphi)}{1 - G\left(\varphi_d^A\right)} &= wf_e, \\
\left[1 - G\left(\varphi_d^A\right)\right]\int_{\varphi_d^A}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1}\frac{r\left(\varphi_d^A\right)}{\sigma} - wf_d\right]\frac{\mathrm{d}G(\varphi)}{1 - G\left(\varphi_d^A\right)} &= wf_e, \\
f_d\int_{\varphi_d^A}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1} - 1\right]\mathrm{d}G(\varphi) &= f_e,
\end{aligned}$$

[1] Following most of the subsequent international trade literature, including Arkolakis, Costinot and Rodriguez-Clare (2012), we consider a static version of Melitz (2003) in which there is zero probability of firm death.

which can be written more compactly as:

$$f_d J\left(\varphi_d^A\right) = f_e, \tag{6}$$

$$J\left(\varphi_d^A\right) = \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^A}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) = \left[1 - G(\varphi_d^A)\right] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1} - 1\right], \tag{7}$$

where $\tilde{\varphi}_d^A$ is a weighted average of firm productivities that corresponds to a harmonic mean weighted by output shares:

$$\tilde{\varphi}_d^A = \left[\int_{\varphi_d^A}^{\varphi_{\max}} \varphi^{\sigma-1} \frac{\mathrm{d}G\left(\varphi\right)}{1 - G\left(\varphi_d^A\right)}\right]^{\frac{1}{\sigma-1}}. \tag{8}$$

Note that $\lim_{\varphi_d^A \to 0} J\left(\varphi_d^A\right) = \infty$, $\lim_{\varphi_d^A \to \infty} J\left(\varphi_d^A\right) = 0$, and $J\left(\varphi_d^A\right)$ is a monotonically decreasing function. It follows that the free entry condition (6) determines a unique equilibrium value of the autarkic zero-profit productivity φ_d^A independently of the other endogenous variables of the model.

The mass of firms (M) equals the mass of entrants (M_e) times the probability of successful entry $(1 - G(\varphi_d^A))$:

$$M = \left[1 - G(\varphi_d^A)\right] M_e = \frac{R}{\bar{r}}. \tag{9}$$

Using the relationship between average firm revenue ($\bar{r}$) and average firm profits ($\bar{\pi}$):

$$\bar{r} = \sigma\left(\bar{\pi} + w f_d\right),$$

and the free entry condition (5), the mass of firms can be re-expressed as:

$$M = \frac{R}{\sigma w \left[\frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d\right]} \tag{10}$$

We choose labor as the numeraire ($w = 1$). Using the relationship between the mass of entrants and mass of firms (9) in the free entry condition (5), we obtain:

$$M\bar{\pi} = M_e f_e = L_e,$$

which implies that total payments to labor used in entry equal total profits. Note that total payments to labor used in production equal total revenue minus total profits:

$$L_p = L - M\bar{\pi},$$

which together with labor market clearing implies that aggregate revenue equals total labor payments ($R = wL = L$). Using this result, the mass of firms (10) can be expressed as:

$$M = \frac{L}{\sigma\left[\frac{f_e}{1 - G\left(\varphi_d^A\right)} + f_d\right]} = \frac{L}{\sigma F^A}, \tag{11}$$

where F^A summarizes average fixed costs per firm in the closed economy. The CES price index in the closed economy can be written as:

$$P = \left[M p\left(\tilde{\varphi}_d^A\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Using the mass of firms (11) and the pricing rule (2), the CES price index becomes:

$$P = \frac{\sigma}{\sigma - 1}\left\{\frac{L}{\sigma F^A}\left(\tilde{\varphi}_d^A\right)^{\sigma-1}\right\}^{\frac{1}{1-\sigma}}.$$

Therefore, using our choice of numeraire, closed economy welfare can be written in terms of the mass of firms ($L/\sigma F^A$) and the weighted average productivity of these firms ($\tilde{\varphi}_d^A$):

$$\mathbb{W}_{\text{Het}}^A = \frac{w}{P} = \frac{\sigma-1}{\sigma}\left\{\frac{L}{\sigma F^A}\left(\tilde{\varphi}_d^A\right)^{\sigma-1}\right\}^{\frac{1}{\sigma-1}}. \tag{12}$$

From the zero-profit condition (4) and the equality of aggregate revenue and total labor payments, welfare can be equivalently written solely in terms of the zero-profit productivity (φ_d^A) and parameters:

$$\mathbb{W}_{\text{Het}}^A = \frac{w}{P} = \left(\frac{L}{\sigma f_d}\right)^{\frac{1}{\sigma-1}}\frac{\sigma-1}{\sigma}\varphi_d^A. \tag{13}$$

Therefore the zero-profit productivity cutoff is a sufficient statistic for welfare.

2.2 Open Economy Heterogeneous Firm Model

We consider trade between two symmetric countries. We assume that there is a fixed exporting cost of f_x units of labor and an iceberg variable trade cost, where $\tau > 1$ units of a variety must be shipped from one country in order for one unit to arrive in the other country. Equilibrium firm revenues in the domestic and export markets are:

$$r_d(\varphi) = RP^{\sigma-1}p_d(\varphi)^{1-\sigma}, \qquad r_x(\varphi) = \tau^{1-\sigma}r_d(\varphi),$$

where the subscript d indicates the domestic market and the subscript x indicates the export market.

Profit maximization implies that equilibrium prices are again a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices because of the variable costs of trade:

$$p_d(\varphi) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}, \qquad p_x(\varphi) = \tau p_d(\varphi), \tag{14}$$

This equilibrium pricing rule implies that profits in each market are a constant proportion of revenues minus the fixed costs:

$$\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - wf_d, \qquad \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - wf_x,$$

where we assume that fixed exporting costs are incurred in the source country and we apportion the fixed production cost to the domestic market. The productivity cutoffs for serving the domestic market (φ_d^T) and export market (φ_x^T) are defined by the following zero-profit conditions:

$$r_d(\varphi_d^T) = R\left(\frac{\sigma-1}{\sigma}P\varphi_d^T\right)^{\sigma-1}w^{1-\sigma} = \sigma w f_d. \tag{15}$$

$$r_x(\varphi_x^T) = R\left(\frac{\sigma-1}{\sigma}P\varphi_x^T\right)^{\sigma-1}(\tau w)^{1-\sigma} = \sigma w f_x, \tag{16}$$

where the superscript T indicates the open economy equilibrium. Together these two zero-profit conditions imply that the export cutoff is a constant multiple of the domestic cutoff that depends on the fixed and variable costs of trade:

$$\varphi_x^T = \tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}\varphi_d^T. \tag{17}$$

For sufficiently high fixed and variable trade costs ($\tau\left(f_x/f_d\right)^{\frac{1}{\sigma-1}} > 1$), only the most productive firms export, consistent with an extensive empirical literature (see for example the review in Bernard, Jensen, Redding and Schott 2007).

The free entry condition again equates the expected value of entry to the sunk entry cost:

$$\left[1-G\left(\varphi_d^T\right)\right]\bar{\pi}=wf_e. \tag{18}$$

Noting that the relative revenues of firms within the same market depend solely on their relative productivities, and using the domestic cutoff condition (15) and the export cutoff condition (16), the free entry condition can be re-written as follows:

$$\begin{aligned}
\left[1-G\left(\varphi_d^T\right)\right]\left[\begin{array}{c}\int_{\varphi_d^T}^{\varphi_{\max}}\left[\frac{r_d(\varphi)}{\sigma}-wf_d\right]\frac{\mathrm{d}G(\varphi)}{1-G\left(\varphi_d^T\right)}\\+\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}\int_{\varphi_x^T}^{\varphi_{\max}}\left[\frac{r_x(\varphi)}{\sigma}-wf_x\right]\frac{\mathrm{d}G(\varphi)}{1-G\left(\varphi_x^T\right)}\end{array}\right] &= wf_e,\\
\left[1-G\left(\varphi_d^T\right)\right]\left[\begin{array}{c}\int_{\varphi_d^T}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1}\frac{r_d\left(\varphi_d^T\right)}{\sigma}-wf_d\right]\frac{\mathrm{d}G(\varphi)}{1-G\left(\varphi_d^T\right)}\\+\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}\int_{\varphi_x^T}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1}\frac{r_x\left(\varphi_x^T\right)}{\sigma}-wf_x\right]\frac{\mathrm{d}G(\varphi)}{1-G\left(\varphi_x^T\right)}\end{array}\right] &= wf_e,\\
\left[\begin{array}{c}f_d\int_{\varphi_d^T}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1}-1\right]\mathrm{d}G\left(\varphi\right)\\+f_x\int_{\varphi_x^T}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1}-1\right]\mathrm{d}G\left(\varphi\right)\end{array}\right] &= f_e.
\end{aligned}$$

or equivalently:

$$f_dJ\left(\varphi_d^T\right)+f_xJ\left(\varphi_x^T\right)=f_e, \tag{19}$$

where $J\left(\cdot\right)$ is defined in (7) and weighted average productivity in the export market $(\tilde{\varphi}_x^T)$ is defined in an analogous way to weighted average productivity in the domestic market $(\tilde{\varphi}_d^T)$ in (8):

$$\tilde{\varphi}_x^T=\left[\int_{\varphi_x^T}^{\varphi_{\max}}\varphi^{\sigma-1}\frac{\mathrm{d}G\left(\varphi\right)}{1-G\left(\varphi_x^T\right)}\right]^{\frac{1}{\sigma-1}}.$$

Using the relationship between the productivity cutoffs (17), the free entry condition can be written solely in terms of the domestic productivity cutoff:

$$\left[\begin{array}{c}f_d\int_{\varphi_d^T}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1}-1\right]\mathrm{d}G\left(\varphi\right)\\+f_x\int_{\tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d^T}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d^T}\right)^{\sigma-1}-1\right]\mathrm{d}G\left(\varphi\right)\end{array}\right]=f_e. \tag{20}$$

Noting that the left-hand side converges towards infinity as φ_d^T converges towards zero; the left-hand side converges towards zero as φ_d^T converges towards infinity; and the left-hand side is monotonically decreasing in φ_d^T. It follows that the free-entry condition (20) determines a unique equilibrium value of φ_d^T independently of the other endogenous variables of the model. The unique equilibrium value of φ_x^T follows immediately from the relationship between the cutoffs (17). Since the left-hand sides of the closed and open economy free entry conditions ((6) and (20) respectively) are monotonically decreasing in φ_d, it also follows that the domestic cutoff in the open economy is greater than the domestic cutoff in the closed economy $(\varphi_d^T>\varphi_d^A)$.

As in the closed economy, the mass of firms (M) equals the mass of entrants (M_e) times the probability of successful entry $(1-G\left(\varphi_d^T\right))$:

$$M=\left[1-G(\varphi_d^T)\right]M_e=\frac{R}{\bar{r}}. \tag{21}$$

Using the relationship between average firm revenue ($\bar{r}$) and average firm profits ($\bar{\pi}$):

$$\bar{r} = \sigma\left(\bar{\pi} + wf_d + \frac{1 - G(\varphi_x^T)}{1 - G(\varphi_d^T)} wf_x\right),$$

and the free entry condition (18) the mass of firms can be re-expressed as:

$$M = \frac{R}{\sigma w\left[\frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x\right]}.$$

where $\chi = \left[1 - G\left(\varphi_x^T\right)\right] / \left[1 - G\left(\varphi_d^T\right)\right]$ is the proportion of exporting firms. As in the closed economy, aggregate revenue equals total labor payments ($R = wL$). We choose labor in one country as the numeraire, which with symmetric countries implies that the wage in each country is equal to one ($w = 1$). Therefore the mass of firms can be written as:

$$M = \frac{L}{\sigma\left[\frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x\right]} = \frac{L}{\sigma F^T}, \tag{22}$$

where F^T summarizes average fixed costs per firm in the open economy. Using the equilibrium pricing rule (14) and our choice of numeraire, the CES price index in the open economy can be written as:

$$P = \frac{\sigma}{\sigma - 1}\left[M\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]\right]^{\frac{1}{1-\sigma}}. \tag{23}$$

Using the price index (23), the mass of firms (22) and our choice of numeraire, open economy welfare can be written in terms of the mass of varieties available for consumption ($L(1+\chi)/\sigma F^T$) and the weighted average productivity of these varieties ($\tilde{\varphi}_t^T$):

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \frac{\sigma - 1}{\sigma}\left\{\frac{L(1+\chi)}{\sigma F^T}\left(\tilde{\varphi}_t^T\right)^{\sigma-1}\right\}^{\frac{1}{\sigma-1}}, \tag{24}$$

where this weighted average productivity ($\tilde{\varphi}_t^T$) depends on weighted average productivity in the domestic and export markets ($\tilde{\varphi}_d^T$ and $\tilde{\varphi}_x^T$ respectively) and the proportion of exporting firms (χ):

$$\left(\tilde{\varphi}_t^T\right)^{\sigma-1} = \frac{1}{1+\chi}\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\left(\tau^{-1}\tilde{\varphi}_x^T\right)^{\sigma-1}\right]. \tag{25}$$

In an open economy equilibrium with selection into export markets ($\varphi_x^T > \varphi_d^T$), the zero-profit condition for the domestic market (15) implies that open economy welfare can be written equivalently in terms of the domestic productivity cutoff and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{L}{\sigma f_d}\right)^{\frac{1}{\sigma-1}}\frac{\sigma - 1}{\sigma}\varphi_d^T. \tag{26}$$

Comparing (13) and (26), and noting that the domestic cutoff is higher in the open economy than in the closed economy ($\varphi_d^T > \varphi_d^A$), there are necessarily welfare gains from trade.

In contrast, in an open economy equilibrium in which all firms export, the domestic and export productivity cutoffs are equal to one another ($\varphi_x^T = \varphi_d^T$), and are determined by the requirement that the sum of variable profits in the domestic and export markets is equal to the sum of fixed production and exporting costs. Using this zero-profit condition, open economy welfare again can be written equivalently in terms of the domestic productivity cutoff and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{\left(1+\tau^{1-\sigma}\right)L}{\sigma\left(f_d + f_x\right)}\right)^{\frac{1}{\sigma-1}}\frac{\sigma - 1}{\sigma}\varphi_d^T. \tag{27}$$

Comparing (13) and (27), and noting that $f_x/f_d \leq \tau^{1-\sigma}$ in an open economy equilibrium in which all firms export, there are again necessarily welfare gains from trade.

2.3 Closed Economy Homogeneous Firm Model

We construct a homogeneous firm model that is a special case of the heterogeneous firm model with a degenerate productivity distribution. Firms pay a sunk entry cost of f_e units of labor and draw a productivity of either zero or $\bar{\varphi}_d$ with exogenous probabilities $\bar{G}_d$ and $\left[1-\bar{G}_d\right]$ respectively. Fixed production costs imply that only firms drawing a productivity of $\bar{\varphi}_d$ find it profitable to produce. Therefore producing firms are homogeneous and there is a degenerate productivity distribution conditional on production at $\bar{\varphi}_d$.

The closed economy equilibrium of this homogeneous firm model is isomorphic to that in Krugman (1980), in which the representative firm's productivity is set equal to $\bar{\varphi}_d$ and the fixed production cost is scaled to incorporate the expected value of entry costs ($\bar{F}_d = f_d + f_e / \left[1-\bar{G}_d\right]$). These values for the representative firm's productivity and the fixed production cost are exogenous and held constant. To simplify the exposition, we adopt this Krugman (1980) interpretation. The representative firm's production technology is:

$$l = \frac{q}{\bar{\varphi}_d} + \bar{F}_d. \tag{28}$$

Consumers again have constant elasticity of substitution (CES) preferences defined over the differentiated varieties supplied by firms. Profit maximization implies that equilibrium prices are a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma-1}\frac{w}{\bar{\varphi}_d}.$$

while profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to the fixed production cost:

$$q = \bar{\varphi}_d \bar{F}_d (\sigma - 1), \qquad l = \sigma \bar{F}_d.$$

Using the common employment for each variety, the mass of firms can be determined from the labor market clearing condition:

$$M = \frac{L}{\sigma \bar{F}_d}. \tag{29}$$

Using the equilibrium pricing rule and the mass of firms, the CES price index is:

$$P^{1-\sigma} = M \left(\frac{\sigma}{\sigma-1}\frac{w}{\bar{\varphi}_d} \right)^{1-\sigma}, \tag{30}$$

where we again choose labor as the numeraire and hence $w = 1$.

Rearranging the price index (30), and using the mass of firms (29) and our choice of numeraire, closed economy welfare can be written in terms of the mass of firms ($L/\sigma\bar{F}_d$) and productivity ($\bar{\varphi}_d$):

$$\mathbb{W}^{A}_{\text{Hom}} = \frac{w}{P} = \frac{\sigma-1}{\sigma} \left\{ \frac{L}{\sigma \bar{F}_d} \left(\bar{\varphi}_d \right)^{\sigma-1} \right\}^{\frac{1}{\sigma-1}}. \tag{31}$$

2.4 Open Economy Homogeneous Firm Model

We again consider trade between two symmetric countries and assume the same trading technology as in the heterogeneous firm model, so that there is a fixed exporting cost of f_x units of labor and an iceberg variable trade cost of $\tau > 1$ units of each variety.

In the homogeneous firm model, the probability of successful entry and productivity conditional on successful entry are exogenous and remain unchanged and equal to $\left[1-\bar{G}_d\right]$ and $\bar{\varphi}_d$ respectively. For sufficiently high fixed and variable trade costs ($\tau^{\sigma-1} f_x / \bar{F}_d > 1$), the representative firm does not find it profitable to export. In contrast, for sufficiently low fixed and variable trade costs ($\tau^{\sigma-1} f_x / \bar{F}_d < 1$), the representative firm finds it profitable to export, and there is trade in both models. The open economy equilibrium of this homogeneous firm model is isomorphic to a version of Krugman (1980) with the same trading technology as in Melitz (2003).

Profit maximization again implies that equilibrium prices are a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices because of the variable costs of trade:

$$p_d = \frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d}, \qquad p_x = \tau p_d. \tag{32}$$

Free entry implies that the representative firm's operating profits equal its fixed costs. In an equilibrium in which the representative firm does not find it profitable to export, we have:

$$\frac{r_d\left(\bar{\varphi}_d\right)}{\sigma} = \bar{F}_d. \tag{33}$$

In contrast, in an equilibrium in which the representative firm finds it profitable to export, we have:

$$\frac{\left(1 + \tau^{1-\sigma}\right) r_d\left(\bar{\varphi}_d\right)}{\sigma} = \bar{F}_d + f_x. \tag{34}$$

Using these two free entry conditions (33) and (34), we can confirm that the representative firm exports if:

$$\tau^{\sigma-1} \frac{f_x}{\bar{F}_d} < 1, \qquad \bar{F}_d = f_d + \frac{f_e}{1 - \bar{G}_d}. \tag{35}$$

In an equilibrium in which the representative firm exports, profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to fixed costs:

$$q = \bar{\varphi}_d \left(\bar{F}_d + f_x\right) (\sigma - 1),$$

$$l = \sigma \left(\bar{F}_d + f_x\right).$$

Therefore both output and employment rise for the representative firm following the opening of trade to cover the additional fixed costs of exporting.

From the labor market clearing condition, this rise in employment for the representative firm implies a fall in the mass of domestically-produced varieties:

$$M = \frac{L}{\sigma\left(\bar{F}_d + f_x\right)}. \tag{36}$$

Using the equilibrium pricing rule and the mass of firms, the CES price index in the open economy is:

$$P^{1-\sigma} = \left[1 + \tau^{1-\sigma}\right] M \left(\frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d}\right)^{1-\sigma}, \tag{37}$$

where we again choose labor as the numeraire and hence $w = 1$.

Using the price index (37), the mass of firms (36) and our choice of numeraire, open economy welfare can be written in terms of the mass of varieties available for consumption $(2L/\sigma(\bar{F}_d + f_x))$ and average productivity $(\bar{\varphi}_t)$:

$$\mathbb{W}^T_{\text{Hom}} = \frac{w}{P} = \frac{\sigma - 1}{\sigma} \left\{ \frac{2L}{\sigma\left(\bar{F}_d + f_x\right)} \left(\bar{\varphi}_t\right)^{\sigma-1} \right\}^{\frac{1}{\sigma-1}}. \tag{38}$$

where this average productivity $(\bar{\varphi}_t)$ depends on the productivity of the representative firm $(\bar{\varphi}_d)$ and variable trade costs (τ):

$$\left(\bar{\varphi}_t\right)^{\sigma-1} = \frac{1}{2} \left[\left(\bar{\varphi}_d\right)^{\sigma-1} + \left(\tau^{-1}\bar{\varphi}_d\right)^{\sigma-1}\right]. \tag{39}$$

3 Theoretical Comparative Static

To examine the aggregate welfare properties of the two models, we first pick the parameters $\bar{G}_d$ and $\bar{\varphi}_d$ of the degenerate productivity distribution with homogeneous firms such that welfare in an initial equilibrium is the same as with heterogeneous firms. We next examine the effects of changes in trade costs. This calibration enables us to undertake a theoretical comparative static in which we examine the effect of the productivity distribution on the model's welfare properties keeping all other structural parameters the same (same $f_d, f_e,\ f_x,\ \tau,\ L,\ \sigma$). We examine both the opening of the closed economy to trade and changes in trade costs in the open economy equilibrium.

3.1 Opening the Closed Economy to Trade

We begin by picking the parameters $\bar{G}_d$ and $\bar{\varphi}_d$ of the degenerate productivity distribution with homogeneous firms such that the autarky equilibrium is isomorphic to that with heterogeneous firms, and examine the effect of opening the closed economy to trade.

Proposition 1 *Consider a homogeneous firm model that is a special case of the heterogeneous firm model with an exogenous probability of successful entry $\left[1-\bar{G}_d\right] = \left[1-G(\varphi_d^A)\right]$ and an exogenous degenerate distribution of productivity conditional on successful entry $\bar{\varphi}_d = \tilde{\varphi}_d^A$. Given the same value for all remaining parameters $\{f_d,\ f_e,\ L,\ \sigma\}$, all aggregate variables (welfare, wage, price index, mass of firms, and aggregate revenue) are the same in the closed economy equilibria of the two models.*

Proof. Comparing (13) and (31), equal welfare follows immediately from $\bar{\varphi}_d = \tilde{\varphi}_d^A$ and $\left[1-\bar{G}_d\right] = \left[1-G(\varphi_d^A)\right]$, which implies $\bar{F}_d = F^A$. Equal wages follow from our choice of numeraire ($w = 1$). Equal welfare and equal wages in turn imply equal price indices. Equal masses of firms follow immediately from equal price indices and $\bar{\varphi}_d = \tilde{\varphi}_d^A$. Equal aggregate revenue follows from $R = wL = L$ in both models. ■

This first proposition reflects the aggregation properties of the heterogeneous firm model. All aggregate variables in this model take the same value as if there were a representative firm with productivity $\bar{\varphi}_d$ and fixed costs $\bar{F}_d$. But the key difference between the heterogeneous firm model and such a representative firm model is that aggregate productivity in the heterogeneous firm model is endogenous and responds to changes in trade costs. From the expressions for open economy welfare in an equilibrium with trade in both models ((24) and (38)), open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model if the following inequality is satisfied:

$$\frac{\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x} > \frac{\left(1+\tau^{1-\sigma}\right)\left(\bar{\varphi}_d\right)^{\sigma-1}}{\bar{F}_d + f_x}.$$

From the free entry condition in the open economy equilibrium of the heterogeneous firm model, and noting that $\bar{F}_d = f_d + f_e/[1-G(\varphi_d^A)]$ and $\bar{\varphi}_d = \tilde{\varphi}_d^A$, this inequality is necessarily satisfied in any open economy equilibrium in the heterogeneous firm model in which the productivity cutoffs differ in the open and closed economies ($\varphi_d^T \neq \varphi_d^A$ and $\varphi_d^T \leq \varphi_x^T \leq \varphi_{\max}$). Since the two models have the same closed economy welfare, it follows that the welfare gains from opening the closed economy to trade are larger in the heterogeneous firm model than in the homogeneous firm model.

Proposition 2 *Choosing the degenerate productivity distribution in the homogeneous firm model so that the two models have the same closed economy welfare and the same structural parameters (f_d, f_e, $f_x,\ \tau,\ L,\ \sigma$), the proportional welfare gains from opening the closed economy to trade are strictly larger in the heterogeneous firm model than in the homogeneous firm model ($\mathbb{W}_{Het}^T/\mathbb{W}_{Het}^A > \mathbb{W}_{Hom}^T/\mathbb{W}_{Hom}^A$), except in the special case with no fixed exporting cost. In this special case, the proportional welfare gains from opening the closed economy to trade are the same in the two models.*

Proof. We establish the proposition for the various possible types of open economy equilibria depending on parameter values. **(I)** First, we consider parameter values for which the representative firm does not find it profitable to export in the homogeneous firm model ($\tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} > 1$). For these parameter values, the proposition follows immediately from the fact that the two models have the same closed economy welfare, there are welfare gains from trade, and trade only occurs in the heterogeneous firm model. **(II)** Second, we consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). From (24) and (38), open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model if the following inequality is satisfied:

$$\frac{\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x} > \frac{\left(1+\tau^{1-\sigma}\right)\left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\bar{F}_d + f_x}. \tag{40}$$

To show that this inequality must be satisfied, we use the open economy free entry condition in the heterogeneous firm model, which implies:

$$f_d \int_{\varphi_d^T}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) + f_x \int_{\varphi_x^T}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) = f_e,$$

$$f_d\left[1-G\left(\varphi_d^T\right)\right]\left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} - 1\right] + f_x\left[1-G\left(\varphi_x^T\right)\right]\left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} - 1\right] = f_e,$$

$$f_d\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1} + f_x\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1} = \frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x.$$

Using $\left(\varphi_x^T\right)^{\sigma-1} = \left(\varphi_d^T\right)^{\sigma-1}\tau^{\sigma-1}f_x/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right] = \frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x. \tag{41}$$

Note that the open economy free entry condition in the heterogeneous firm model also implies:

$$f_d \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) + f_x \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) < f_e, \tag{42}$$

since $\varphi_d^A < \varphi_d^T < \varphi_x^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^T,$$

$$\left[\left(\frac{\varphi}{\varphi_x^T}\right)^{\sigma-1} - 1\right] < 0 \qquad \text{for} \qquad \varphi < \varphi_x^T.$$

Rewriting (42), we have:

$$f_d\left[1-G\left(\varphi_d^A\right)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T}\right)^{\sigma-1} - 1\right] + f_x\left[1-G\left(\varphi_d^A\right)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T}\right)^{\sigma-1} - 1\right] < f_e,$$

$$f_d\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T}\right)^{\sigma-1} + f_x\left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T}\right)^{\sigma-1} < \frac{f_e}{1-G\left(\varphi_d^A\right)} + f_d + f_x.$$

Using $\left(\varphi_x^T\right)^{\sigma-1} = \left(\varphi_d^T\right)^{\sigma-1} \tau^{\sigma-1} f_x/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1+\tau^{1-\sigma}\right) \left(\tilde{\varphi}_d^A\right)^{\sigma-1} < \frac{f_e}{1-G\left(\varphi_d^A\right)} + f_d + f_x. \tag{43}$$

From (41) and (43), we have:

$$\frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]}{\frac{f_e}{1-G\left(\varphi_d^T\right)} + f_d + \chi f_x} = 1, \tag{44}$$

$$\frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1+\tau^{1-\sigma}\right)\left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi_d^A\right)} + f_d + f_x} = \frac{\frac{f_d}{\left(\varphi_d^T\right)^{\sigma-1}} \left(1+\tau^{1-\sigma}\right)\left(\tilde{\varphi}_d^A\right)^{\sigma-1}}{\bar{F}_d + f_x} < 1,$$

which establishes that inequality (40) is satisfied. From (26) and (38), the condition for open economy welfare to be higher in the heterogeneous firm model with export market selection than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1+\tau^{1-\sigma}}{\bar{F}_d + f_x}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

Using (40) and (44), this (equivalent) inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}_{\text{Het}}^T/\mathbb{W}_{\text{Het}}^A > \mathbb{W}_{\text{Hom}}^T/\mathbb{W}_{\text{Hom}}^A$). **(III)** Third, we consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < \tau\left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$). This is simply a special case of **(II)** in which $\varphi_x^T = \varphi_d^T$, $\tilde{\varphi}_x^T = \tilde{\varphi}_d^T$ and $\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)} = 1$. Therefore the same line of reasoning as in **(II)** can be used to show that the inequality (40) is satisfied and hence open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model. In this special case in which all firms export, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$\left(f_d + f_x\right) \int_{\varphi_d^T}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G\left(\varphi\right) = f_e,$$

$$\left(f_d + f_x\right) \int_{\varphi_d^A}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] \mathrm{d}G\left(\varphi\right) < f_e, \tag{45}$$

since $\varphi_d^A < \varphi_d^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1\right] < 0, \qquad \text{for} \qquad \varphi < \varphi_d^T.$$

Rewriting (45), we obtain:

$$\left(f_d + f_x\right) \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T}\right)^{\sigma-1} < \frac{f_e}{1-G\left(\varphi_d^A\right)} + f_d + f_x = \bar{F}_d + f_x. \tag{46}$$

From (27) and (38), the condition for open economy welfare to be higher in the heterogeneous firm model without export market selection than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d + f_x}\right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1}{\bar{F}_d + f_x}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

From (46), this inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}^T_{\text{Het}}/\mathbb{W}^A_{\text{Het}} > \mathbb{W}^T_{\text{Hom}}/\mathbb{W}^A_{\text{Hom}}$). **(IV)** Finally, when fixed exporting costs are zero, we have $0 = \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} = \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$. This is a special case of **(III)** in which $\varphi^T_x = \varphi^T_d = \varphi^A_d$, $\tilde{\varphi}^T_x = \tilde{\varphi}^T_d = \tilde{\varphi}^A_d$ and $\frac{1-G(\varphi^T_x)}{1-G(\varphi^T_d)} = 1$. In this special case of zero fixed exporting costs, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$(f_d + f_x)\int_{\varphi^A_d}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^T_d}\right)^{\sigma-1} - 1\right] \mathrm{d}G(\varphi) = f_e,$$

$$(f_d + f_x)\left(\frac{\tilde{\varphi}^A_d}{\varphi^T_d}\right)^{\sigma-1} = \frac{f_e}{1 - G\left(\varphi^A_d\right)} + f_d + f_x = \bar{F}_d + f_x,$$

where we have used $\varphi^A_d = \varphi^T_d$. From (27) and (38), it follows immediately that open economy welfare is the same in the two models when fixed exporting costs are equal to zero. ■

Since the proportional welfare gains from trade are strictly lower in the homogeneous firm model than in the heterogeneous firm model for positive fixed exporting costs, and since open economy welfare in the homogeneous firm model is monotonically decreasing in trade costs, we also obtain the following result.

Proposition 3 *Achieving the same proportional welfare gains from trade in the two models requires strictly lower trade costs (either lower f_x and/or lower τ) in the homogeneous firm model than in the heterogeneous firm model, except in the special case with no fixed exporting cost.*

Proof. The proposition follows immediately from $\mathbb{W}^T_{\text{Het}}/\mathbb{W}^A_{\text{Het}} > \mathbb{W}^T_{\text{Hom}}/\mathbb{W}^A_{\text{Hom}}$ in Proposition 2 and from $\frac{\mathrm{d}\mathbb{W}^T_{\text{Hom}}}{\mathrm{d}f_x} < 0$ and $\frac{\mathrm{d}\mathbb{W}^T_{\text{Hom}}}{\mathrm{d}\tau} < 0$ in (38). ■

Although we chose the productivity of the representative firm ($\bar{\varphi}_d = \tilde{\varphi}^A_d$) to ensure the same closed economy welfare in both models, the ratio of open to closed economy welfare in the homogeneous firm model $\mathbb{W}^T_{\text{Hom}}/\mathbb{W}^A_{\text{Hom}}$ is independent of the representative firm's productivity (from (31) and (38)). It follows that both of the above propositions hold for any value of the representative firm's productivity. Both these propositions also hold for general continuous productivity distributions.

3.2 Changes in Trade Costs in the Open Economy Equilibrium

The role of the additional adjustment margin of firm entry and exit decisions for generating different aggregate welfare implications is not limited to the opening of the closed economy to trade and also holds for reductions in trade costs in the open economy equilibrium. To show this, we recast our heterogeneous and homogeneous firm models so that they have the same welfare in an initial open economy equilibrium. In order to ensure that the two models have the same initial welfare and only differ in their productivity distribution (keeping the same structural parameters f_d, f_e, f_x, τ, L, σ), we extend the homogeneous firm model to allow for two types of firms: exporters and non-exporters. In this extension, firms again pay a sunk entry cost of f_e units of labor before observing their productivity. With probability $\left[1 - \bar{G}_x\right]$ a firm draws a productivity of $\bar{\varphi}_x$ and can export; with probability $\bar{G}_{dx}$ the firm draws a productivity of $\bar{\varphi}_{dx}$ and cannot export; with probability $\left[\bar{G}_x - \bar{G}_{dx}\right]$ the firm draws a productivity of zero and does not find it profitable to produce.

We pick the parameters of this "extended" homogeneous firm model ($\bar{\varphi}_x, \bar{\varphi}_{dx}, \bar{G}_x, \bar{G}_{dx}$) such that the open economy equilibrium features the same aggregate variables as the initial open economy equilibrium with heterogeneous firms (same welfare, price index, mass of firms, aggregate revenue, and domestic trade share). This requires equating those parameters with their corresponding averages under firm heterogeneity:

$$\bar{G}_x = G\left(\varphi_x^T\right), \qquad \bar{G}_{dx} = G\left(\varphi_x^T\right) - G\left(\varphi_d^T\right),$$

$$\bar{\varphi}_x = \tilde{\varphi}_x^T, \qquad \bar{\varphi}_{dx} = \left\{\frac{1}{G\left(\varphi_x^T\right) - G\left(\varphi_d^T\right)} \int_{\varphi_d^T}^{\varphi_x^T} \varphi^{\sigma-1} \mathrm{d}G\left(\varphi\right)\right\}^{\frac{1}{\sigma-1}},$$

where the last term represents the average productivity of non-exporters in the heterogeneous firm model. We keep the values of all the other structural parameters (f_d, f_e, f_x, τ, L, σ) constant across the two models. This ensures that the aggregate statistics line-up across the two models in the initial open economy equilibrium.

Nevertheless these two models respond differently to changes in trade costs from this common initial equilibrium along a key dimension. In the heterogeneous firm model, the endogenous selection responses to trade costs lead to changes in the average productivity of exporting and non-exporting firms and in the proportion of exporting firms. In contrast, in the extended homogeneous firm model, the average productivity levels of exporters and non-exporters and the proportion of exporting firms remain constant.[2] The presence of this additional adjustment margin in the heterogeneous firm model implies that welfare following the change in trade costs must be strictly higher than in the homogeneous from model. This argument holds irrespective of whether trade costs decrease or increase. Therefore, welfare *gains* are *larger* in the heterogeneous firm model whenever trade costs fall, and welfare *losses* are *smaller* in the heterogeneous firm model whenever trade costs *increase*.

Proposition 4 *Starting from an initial open economy equilibrium with the same welfare and the same structural parameters in the two models (f_d, f_e, f_x, τ, L, σ), a common decrease (increase) in variable or fixed trade costs generates larger welfare gains (smaller welfare losses) in the heterogeneous firm model than in the extended homogeneous firm model.*

Proof. In the initial open economy equilibrium before the change in trade costs, (24) implies that welfare in both the heterogeneous firm model and in the extended homogeneous firm model can be written as:

$$\left(\mathbb{W}_{\text{Het}}^{T_1}\right)^{\sigma-1} = \frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\tilde{\varphi}_d^{T_1}\right)^{\sigma-1} + \chi_1 \tau_1^{1-\sigma}\left(\tilde{\varphi}_x^{T_1}\right)^{\sigma-1}\right]}{\sigma\left[\frac{f_e}{1-G\left(\varphi_d^{T_1}\right)} + f_d + \chi_1 f_{x1}\right]}.$$

In the new open economy equilibrium after the change in trade costs, (24) implies that welfare in the heterogeneous firm model is:

$$\left(\mathbb{W}_{\text{Het}}^{T_2}\right)^{\sigma-1} = \frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\tilde{\varphi}_d^{T_2}\right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_2}\right)^{\sigma-1}\right]}{\sigma\left[\frac{f_e}{1-G\left(\varphi_d^{T_2}\right)} + f_d + \chi_2 f_{x2}\right]}.$$

In contrast, in the new open economy equilibrium after the change in trade costs, welfare in the

[2] Unless trade costs become sufficiently high that firms with productivity $\bar{\varphi}_x$ no longer find it profitable to export or firms with productivity $\bar{\varphi}_{dx}$ no longer find it profitable to produce. In both cases, the average productivity of the two types of firms remains constant at $\bar{\varphi}_x$ and $\bar{\varphi}_{dx}$

extended homogeneous firm model is:

$$\left(\mathbb{W}^{T_2}_{\text{Hom}}\right)^{\sigma-1} = \frac{L\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\tilde{\varphi}^{T_1}_d\right)^{\sigma-1} + \chi_1\tau_2^{1-\sigma}\left(\tilde{\varphi}^{T_1}_x\right)^{\sigma-1}\right]}{\sigma\left[\frac{f_e}{1-G\left(\varphi^{T_1}_d\right)} + f_d + \chi_1 f_{x2}\right]}$$

To show that welfare in the new open economy equilibrium is higher in the heterogeneous firm model than in the homogeneous firm model, we need to show that:

$$\frac{\left(\tilde{\varphi}^{T_2}_d\right)^{\sigma-1} + \chi_2\tau_2^{1-\sigma}\left(\tilde{\varphi}^{T_2}_x\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi^{T_2}_d\right)} + f_d + \chi_2 f_{x2}} > \frac{\left(\tilde{\varphi}^{T_1}_d\right)^{\sigma-1} + \chi_1\tau_2^{1-\sigma}\left(\tilde{\varphi}^{T_1}_x\right)^{\sigma-1}}{\frac{f_e}{1-G\left(\varphi^{T_1}_d\right)} + f_d + \chi_1 f_{x2}}. \tag{47}$$

To establish this inequality, we use the free entry condition in the new open economy equilibrium of the heterogeneous firm model, which implies:

$$f_d\int_{\varphi^{T_2}_d}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi^{T_2}_d}\right)^{\sigma-1} - 1\right]\mathrm{d}G\left(\varphi\right) + f_{x2}\int_{\varphi^{T_2}_x}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi^{T_2}_x}\right)^{\sigma-1} - 1\right]\mathrm{d}G\left(\varphi\right) = f_e,$$

$$f_d\left[1 - G\left(\varphi^{T_2}_d\right)\right]\left[\left(\frac{\tilde{\varphi}^{T_2}_d}{\varphi^{T_2}_d}\right)^{\sigma-1} - 1\right] + f_{x2}\left[1 - G\left(\varphi^{T_2}_x\right)\right]\left[\left(\frac{\tilde{\varphi}^{T_2}_x}{\varphi^{T_2}_x}\right)^{\sigma-1} - 1\right] = f_e,$$

$$f_d\left(\frac{\tilde{\varphi}^{T_2}_d}{\varphi^{T_2}_d}\right)^{\sigma-1} + f_{x2}\frac{1 - G\left(\varphi^{T_2}_x\right)}{1 - G\left(\varphi^{T_2}_d\right)}\left(\frac{\tilde{\varphi}^{T_2}_x}{\varphi^{T_2}_x}\right)^{\sigma-1} = \frac{f_e}{1 - G\left(\varphi^{T_2}_d\right)} + f_d + \frac{1 - G\left(\varphi^{T_2}_x\right)}{1 - G\left(\varphi^{T_2}_d\right)}f_{x2}.$$

Using $\left(\varphi^{T_2}_x\right)^{\sigma-1} = \left(\varphi^{T_2}_d\right)^{\sigma-1}\tau_2^{\sigma-1}f_{x2}/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi^{T_2}_d\right)^{\sigma-1}}\left[\left(\tilde{\varphi}^{T_2}_d\right)^{\sigma-1} + \chi_2\tau_2^{1-\sigma}\left(\tilde{\varphi}^{T_2}_x\right)^{\sigma-1}\right] = \frac{f_e}{1 - G\left(\varphi^{T_2}_d\right)} + f_d + \chi_2 f_{x2}. \tag{48}$$

Note that the free entry condition in the new open economy equilibrium of the heterogeneous firm model also implies:

$$f_d\int_{\varphi^{T_1}_d}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi^{T_2}_d}\right)^{\sigma-1} - 1\right]\mathrm{d}G\left(\varphi\right) + f_{x2}\int_{\varphi^{T_1}_x}^{\varphi_{\max}}\left[\left(\frac{\varphi}{\varphi^{T_2}_x}\right)^{\sigma-1} - 1\right]\mathrm{d}G\left(\varphi\right) < f_e, \tag{49}$$

since $\varphi^{T_1}_d < \varphi^{T_2}_d$ and $\varphi^{T_1}_x > \varphi^{T_2}_x$ and

$$\left[\left(\frac{\varphi}{\varphi^{T_2}_d}\right)^{\sigma-1} - 1\right] < 0, \qquad \text{for} \qquad \varphi < \varphi^{T_2}_d,$$

$$\left[\left(\frac{\varphi}{\varphi^{T_2}_x}\right)^{\sigma-1} - 1\right] > 0 \qquad \text{for} \qquad \varphi^{T_2}_x < \varphi < \varphi^{T_1}_x.$$

Rewriting (49), we have:

$$f_d\left[1 - G\left(\varphi^{T_1}_d\right)\right]\left[\left(\frac{\tilde{\varphi}^{T_1}_d}{\varphi^{T_2}_d}\right)^{\sigma-1} - 1\right] + f_{x_2}\left[1 - G\left(\varphi^{T_1}_x\right)\right]\left[\left(\frac{\tilde{\varphi}^{T_1}_x}{\varphi^{T_2}_x}\right)^{\sigma-1} - 1\right] < f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^{T_1}}{\varphi_d^{T_2}}\right)^{\sigma-1} + f_{x2}\frac{1-G\left(\varphi_x^{T_1}\right)}{1-G\left(\varphi_d^{T_1}\right)}\left(\frac{\tilde{\varphi}_x^{T_1}}{\varphi_x^{T_2}}\right)^{\sigma-1} < \frac{f_e}{1-G\left(\varphi_d^{T_1}\right)} + f_d + \frac{1-G\left(\varphi_x^{T_1}\right)}{1-G\left(\varphi_d^{T_1}\right)} f_{x2}.$$

Using $\left(\varphi_x^{T_2}\right)^{\sigma-1} = \left(\varphi_d^{T_2}\right)^{\sigma-1} \tau_2^{\sigma-1} f_{x2}/f_d$, we obtain:

$$\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^{T_1}\right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_1}\right)^{\sigma-1}\right] < \frac{f_e}{1-G\left(\varphi_d^{T_1}\right)} + f_d + \chi_1 f_{x2}. \tag{50}$$

From (48) and (50), we have:

$$\frac{\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^{T_2}\right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_2}\right)^{\sigma-1}\right]}{\frac{f_e}{1-G\left(\varphi_d^{T_2}\right)} + f_d + \chi_2 f_{x2}} = 1,$$

$$\frac{\frac{f_d}{\left(\varphi_d^{T_2}\right)^{\sigma-1}}\left[\left(\tilde{\varphi}_d^{T_1}\right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_1}\right)^{\sigma-1}\right]}{\frac{f_e}{1-G\left(\varphi_d^{T_1}\right)} + f_d + \chi_1 f_{x2}} < 1,$$

which establishes the inequality (47). ∎

Note that the extended homogeneous firm model is equivalent to a version of the heterogeneous firm model in which the domestic and export productivity cutoffs are held constant at their values in an initial open economy equilibrium. Put another way, consider a planner who is constrained to keep the same set of firms operating in both the domestic and export markets – i.e. the endogenous selection margin is inoperative. Under this constraint, the welfare-maximizing allocation coincides with the market equilibrium of the extended homogeneous firm model. In contrast, in the absence of this constraint, the welfare-maximizing allocation coincides with the market equilibrium of the heterogeneous firm model. Therefore, the welfare differential between the two models provides a direct measure of the impact of selection on aggregate welfare. In other words, it isolates the *additional* contribution to aggregate welfare of the new endogenous selection/productivity channel highlighted by the heterogeneous firm model of trade – this represents the new welfare implications that we refer to in the title of this paper. Later in Section 6, we show that this additional welfare channel is quantitatively substantial for a model calibrated to U.S. aggregate and firm statistics.

Atkeson and Burstein (2010) considers this welfare differential from endogenous firm selection in a model with product and process innovation. They find that this welfare differential is of second-order. Proposition 4 is consistent with this result. As discussed above and shown formally in the web appendix, the initial equilibrium of the heterogeneous firm model is efficient. Therefore the envelope theorem implies that the changes in the productivity cutoffs in the heterogeneous firm model have only second-order effects on welfare. But, as we show later, these second-order welfare effects can be quite substantial for larger changes in trade costs.

3.3 Untruncated Pareto Distribution

Since the homogeneous firm model is a special case of the heterogeneous firm model, our above comparisons of the two models are equivalent to a discrete comparative static of moving from a non-degenerate to a degenerate productivity distribution within the heterogeneous firm model. In the special case of an untruncated Pareto productivity distribution, the degree of firm heterogeneity is summarized by a single parameter: the shape parameter k. Lower values of k correspond to greater firm heterogeneity and the homogeneous firm model corresponds to the limiting case in which $k \to \infty$.

Therefore we can complement the above discrete comparative static with a continuous comparative static in the degree of firm heterogeneity.

Proposition 5 *Assuming that productivity in the heterogeneous firm model has an untruncated Pareto distribution ($g(\varphi) = k\varphi_{\min}^{k}\varphi^{-(k+1)}$, where $\varphi \geq \varphi_{\min} > 0$ and $k > \sigma - 1$) and fixed exporting costs are positive, the greater the dispersion of firm productivity (smaller k),* ***(a)*** *the larger the welfare gains from opening the closed economy to trade (larger $\mathbb{W}_{Het}^{T}/\mathbb{W}_{Het}^{A}$),* ***(b)*** *the larger (smaller) the welfare gains (losses) from a decrease (increase) in variable trade costs in the open economy equilibrium.*

Proof. (a) First, consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). From (13) and (26), we have:

$$\frac{\mathbb{W}_{\text{Het}}^{T}}{\mathbb{W}_{\text{Het}}^{A}} = \frac{\varphi_d^T}{\varphi_d^A}. \tag{51}$$

In the special case of an untruncated Pareto productivity distribution and for these parameter values for which there is selection into export markets in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A} = \left[1 + \left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^{k} \frac{f_x}{f_d}\right]^{1/k},$$

which can be written as:

$$\ln\left(\frac{\varphi_d^T}{\varphi_d^A}\right) = k^{-1}\ln\left[1 + \left(\frac{1}{\tau\left(f_x/f_d\right)^{1/(\sigma-1)}}\right)^{k} \frac{f_x}{f_d}\right].$$

Note that

$$\frac{\mathrm{d}\ln\left(\varphi_d^T/\varphi_d^A\right)}{\mathrm{d}k} = -k^{-2}\ln\left[1 + \left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^{k}\frac{f_x}{f_d}\right] - \frac{k^{-1}\ln\left(\tau(f_x/f_d)^{1/(\sigma-1)}\right)\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^{k}\frac{f_x}{f_d}}{\left[1+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^{k}\frac{f_x}{f_d}\right]} < 0, \tag{52}$$

where we have used $\mathrm{d}\left(a^x\right)/\mathrm{d}x = (\ln a)\,a^x$. Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion implies larger φ_d^T/φ_d^A. Second, consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < \tau\left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$). From (13) and (27), we have:

$$\frac{\mathbb{W}_{\text{Het}}^{T}}{\mathbb{W}_{\text{Het}}^{A}} = \left(\frac{\left(1+\tau^{1-\sigma}\right)f_d}{f_d+f_x}\right)^{\frac{1}{\sigma-1}}\frac{\varphi_d^T}{\varphi_d^A}.$$

In the special case of an untruncated Pareto productivity distribution and for these parameter values for which all firms export in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A} = \left[1 + \frac{f_x}{f_d}\right]^{1/k},$$

which can be written as:

$$\ln\left(\frac{\varphi_d^T}{\varphi_d^A}\right) = k^{-1}\ln\left[1 + \frac{f_x}{f_d}\right].$$

Note that

$$\frac{\mathrm{d}\ln\left(\varphi_d^T/\varphi_d^A\right)}{\mathrm{d}k} = -k^{-2}\ln\left[1+\frac{f_x}{f_d}\right] < 0. \tag{53}$$

Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion again implies larger φ_d^T/φ_d^A. Taking (52) and (53) together and using (51), it follows that greater dispersion of firm productivity (smaller k) implies larger proportional welfare gains from opening the closed economy to trade. **(b)** Consider parameter values for which there is selection into export markets in the open economy equilibrium of the heterogeneous firm model ($\tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1$). In the special case of an untruncated Pareto productivity distribution, we have:

$$\varphi_d^T = \left(\frac{\sigma-1}{k-(\sigma-1)}\right)^{1/k}\left[\frac{f_d+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_e}\right]^{1/k}\varphi_{\min}.$$

Therefore:

$$\begin{aligned}\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau &= -\frac{\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_d+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}\mathrm{d}\tau \\ &= -\xi\mathrm{d}\tau.\end{aligned}$$

Hence:

$$\frac{\mathrm{d}\left(\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau\right)}{\mathrm{d}k} = \frac{\ln\left(\tau\left(f_x/f_d\right)^{1/(\sigma-1)}\right)\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}{f_d+\left(\frac{1}{\tau(f_x/f_d)^{1/(\sigma-1)}}\right)^k f_x}\left(1-\xi\right)\mathrm{d}\tau,$$

which implies:

$$\begin{aligned}\frac{\mathrm{d}\left(\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau\right)}{\mathrm{d}k} &< 0 \qquad \text{for} \qquad \mathrm{d}\tau < 0, \\ \frac{\mathrm{d}\left(\frac{\mathrm{d}\varphi_d^T}{\mathrm{d}\tau}\frac{\tau}{\varphi_d^T}\mathrm{d}\tau\right)}{\mathrm{d}k} &> 0 \qquad \text{for} \qquad \mathrm{d}\tau > 0.\end{aligned}$$

Therefore greater dispersion of firm productivity (smaller k) implies a larger elasticity of the domestic productivity cutoff with respect to reductions in variable trade costs, which from (26) implies greater proportional welfare gains from reductions in variable trade costs. By the same reasoning, greater dispersion of firm productivity (smaller k) implies a smaller elasticity of the domestic productivity cutoff with respect to increases in variable trade costs, which from (26) implies smaller proportional welfare costs from increases in variable trade costs. ■

In this special case of an untruncated Pareto distribution, the heterogeneous firm model falls within the class considered by ACR. Therefore Proposition 5 confirms that the degree of firm heterogeneity affects the aggregate welfare implications of trade, in the sense of our theoretical comparative static, even within the class of models considered by ACR.

In the special case of an untruncated Pareto distribution, we obtain the following closed-form solutions for the domestic and export productivity cutoffs in the heterogeneous firm model in terms of the model's parameters:

$$\left(\varphi_d^A\right)^k = \frac{\sigma-1}{k-(\sigma-1)}\frac{f_d}{f_e}\varphi_{\min}^k, \tag{54}$$

$$\left(\varphi_d^T\right)^k = \frac{\sigma-1}{k-(\sigma-1)} \left[\frac{f_d + \tau^{-k}\left(\frac{f_x}{f_d}\right)^{\frac{-k}{\sigma-1}} f_x}{f_e} \right] \varphi_{\min}^k. \tag{55}$$

Using these domestic and export productivity cutoffs, we can obtain closed-form solutions for the relative welfare gains from trade in heterogeneous and homogeneous firm models. We begin by considering the opening of the closed economy to trade, in which case the degenerate productivity distribution in the homogeneous firm model is chosen so that the two models have the same closed economy welfare. Therefore the relative welfare gains from trade in the two models equal relative open economy welfare. From (40), relative open economy welfare in the two models is:

$$\left[\frac{\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\left(1+\tau^{1-\sigma}\right)\left(\tilde{\varphi}_d^A\right)^{\sigma-1}} \right] \left[\frac{\frac{f_e}{1-G(\varphi_d^A)} + f_d + f_x}{\frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x} \right] > 1. \tag{56}$$

In the special case of an untruncated Pareto distribution, this expression for relative welfare becomes:

$$\left[\frac{(\varphi_d^T)^{-(k-(\sigma-1))} + \tau^{1-\sigma}(\varphi_x^T)^{-(k-(\sigma-1))}}{(1+\tau^{1-\sigma})(\varphi_d^A)^{-(k-(\sigma-1))}} \right] \left[\frac{f_e + \left(\frac{\varphi_{\min}}{\varphi_d^A}\right)^k f_d + \left(\frac{\varphi_{\min}}{\varphi_d^A}\right)^k f_x}{f_e + \left(\frac{\varphi_{\min}}{\varphi_d^T}\right)^k f_d + \left(\frac{\varphi_{\min}}{\varphi_x^T}\right)^k f_x} \right] > 1, \tag{57}$$

where $\varphi_x^T = \tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d^T$ and we have closed-form solutions for $\{\varphi_d^A, \varphi_d^T\}$ from (54) and (55).

We next compare two open economy equilibria with different values of trade costs, in which case the productivity distribution in the extended homogeneous firm model is chosen so that the two models have the same welfare in the initial open economy equilibrium (indexed by T_1). Therefore the relative welfare gains from trade liberalization in the two models equal relative welfare in the new open economy equilibrium following the reduction in variable trade costs (indexed by T_2). From (47), relative open economy welfare in the two models following trade liberalization is:

$$\left[\frac{\left(\tilde{\varphi}_d^{T_2}\right)^{\sigma-1} + \chi_2\tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_2}\right)^{\sigma-1}}{\left(\tilde{\varphi}_d^{T_1}\right)^{\sigma-1} + \chi_1\tau_2^{1-\sigma}\left(\tilde{\varphi}_x^{T_1}\right)^{\sigma-1}} \right] \left[\frac{\frac{f_e}{1-G\left(\varphi_d^{T_1}\right)} + f_d + \chi_1 f_{x2}}{\frac{f_e}{1-G\left(\varphi_d^{T_2}\right)} + f_d + \chi_2 f_{x2}} \right] > 1 \tag{58}$$

In the special case of an untruncated Pareto distribution, this expression for relative welfare becomes:

$$\left[\frac{(\varphi_d^{T_2})^{-(k-(\sigma-1))} + \tau_2^{1-\sigma}(\varphi_x^{T_2})^{-(k-(\sigma-1))}}{(\varphi_d^{T_1})^{-(k-(\sigma-1))} + \tau_2^{1-\sigma}(\varphi_x^{T_1})^{-(k-(\sigma-1))}} \right] \left[\frac{f_e + \left(\frac{\varphi_{\min}}{\varphi_d^{T_1}}\right)^k f_d + \left(\frac{\varphi_{\min}}{\varphi_x^{T_1}}\right)^k f_{x2}}{f_e + \left(\frac{\varphi_{\min}}{\varphi_d^{T_2}}\right)^k f_d + \left(\frac{\varphi_{\min}}{\varphi_x^{T_2}}\right)^k f_{x2}} \right] > 1, \tag{59}$$

where $\varphi_x^T = \tau(f_x/f_d)^{1/(\sigma-1)}\varphi_d^T$ in an open economy equilibrium with export market selection and we have closed-form solutions for φ_d^T from (55) above.

4 Welfare and Trade Policy Evaluation

Our theoretical comparative static in the previous section examines the impact of changes in the distribution of productivity holding other exogenous variables fixed across models. This exercise does not restrict the equilibrium values of the endogenous variables (in particular the domestic trade share λ and the trade elasticity θ) to be the same in the two models. Instead the equilibrium values for these endogenous variables differ systematically across the two models. We now compare trade shares and trade elasticities in the two models given the same values of the exogenous variables.

Trade Shares: In an open economy equilibrium of the homogeneous firm model in which the representative firm exports, the domestic trade share is:

$$\lambda_{\text{Hom}} = \frac{\left(\bar{\varphi}_d\right)^{\sigma-1}}{\left(1+\tau^{1-\sigma}\right)\left(\bar{\varphi}_d\right)^{\sigma-1}} = \frac{1}{1+\tau^{1-\sigma}}. \tag{60}$$

In contrast, in an open economy equilibrium of the heterogeneous firm model, the domestic trade share is:

$$\lambda_{\text{Het}} = \frac{\left(\tilde{\varphi}_d^T\right)^{\sigma-1}}{\left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}} = \frac{1}{1+\tau^{1-\sigma}\Lambda}, \tag{61}$$

$$\text{where } \Lambda = \frac{\delta(\varphi_x)}{\delta(\varphi_d)} = \frac{\int_{\varphi_x^T}^{\varphi_{\max}} \varphi^{\sigma-1}\mathrm{d}G\left(\varphi\right)}{\int_{\varphi_d^T}^{\varphi_{\max}} \varphi^{\sigma-1}\mathrm{d}G\left(\varphi\right)}.$$

In an open economy equilibrium of the heterogeneous firm model in which only some firms export ($0 < \varphi_d^T < \varphi_x^T$), the export market selection term Λ is strictly less than one.

Proposition 6 *Given the same structural parameters ($f_d, f_e, f_x, \tau, L, \sigma$), the domestic trade share is strictly greater in the heterogeneous firm model than in the homogeneous firm model ($\lambda_{Het} > \lambda_{Hom}$) for parameter values for which there is trade in both models and selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). The domestic trade share is only the same in the two models ($\lambda_{Het} = \lambda_{Hom}$) for parameter values for which all firms export in the heterogeneous firm model ($0 \leq \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < \tau\left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$).*

Proof. For $0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$, we have $\varphi_x^T > \varphi_d^T$, which implies $0 < \Lambda < 1$ and hence $\lambda_{d,\text{Het}} > \lambda_{d,\text{Hom}}$ in the domestic trade shares (61) and (60). For $0 \leq \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < \tau\left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$, we have $\varphi_x^T = \varphi_d^T$, $\Lambda = 1$ and $\lambda_{\text{Het}} = \lambda_{\text{Hom}}$. ■

In the special case of an untruncated Pareto productivity distribution from Proposition 5, we can solve in closed form for the export market selection term (Λ) in the heterogeneous firm model as a function of the productivity cutoffs $\{\varphi_d^T, \varphi_x^T\}$:

$$\Lambda = \frac{\int_{\varphi_x^T}^{\varphi_{\max}} \varphi^{\sigma-1}\mathrm{d}G\left(\varphi\right)}{\int_{\varphi_d^T}^{\varphi_{\max}} \varphi^{\sigma-1}\mathrm{d}G\left(\varphi\right)} = \left(\frac{\varphi_d^T}{\varphi_x^T}\right)^{k-(\sigma-1)}. \tag{62}$$

In an open economy equilibrium with selection into export markets ($\varphi_d^T < \varphi_x^T$), we have:

$$\lambda_{\text{Het}} = \frac{1}{1+\tau^{-(\sigma-1)}\left(\frac{\varphi_d^T}{\varphi_x^T}\right)^{k-(\sigma-1)}} = \frac{1}{1+\tau^{-k}\left(\frac{f_x}{f_d}\right)^{-\frac{k-(\sigma-1)}{\sigma-1}}} > \lambda_{\text{Hom}} = \frac{1}{1+\tau^{-(\sigma-1)}}. \tag{63}$$

A generalization of this functional form for the productivity distribution is the case of a truncated Pareto distribution, in which the support of the productivity distribution is bounded from above:

$$g(\varphi) = \frac{k\varphi_{\min}^k\varphi^{-(k+1)}}{1-\left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

with the corresponding cumulative distribution function:

$$G(\varphi) = \frac{1-\left(\frac{\varphi_{\min}}{\varphi}\right)^k}{1-\left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k},$$

which implies:

$$1 - G(\varphi) = \frac{\left(\frac{\varphi_{\min}}{\varphi}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}{1 - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

where the untruncated Pareto distribution is the special case in which $\varphi_{\max} \to \infty$. With this more general functional form, the export market selection term (Λ) continues to be a closed-form expression of the productivity cutoffs $\{\varphi_d^T, \varphi_x^T\}$:

$$\Lambda = \frac{\int_{\varphi_x^T}^{\varphi_{\max}} \varphi^{\sigma-1} \mathrm{d}G(\varphi)}{\int_{\varphi_d^T}^{\varphi_{\max}} \varphi^{\sigma-1} \mathrm{d}G(\varphi)} = \left(\frac{\varphi_d^T}{\varphi_x^T}\right)^{k-(\sigma-1)} \frac{1 - \left(\frac{\varphi_x^T}{\varphi_{\max}}\right)^{k-(\sigma-1)}}{1 - \left(\frac{\varphi_d^T}{\varphi_{\max}}\right)^{k-(\sigma-1)}}. \tag{64}$$

In an open economy equilibrium with selection into export markets ($\varphi_d^T < \varphi_x^T$), we have:

$$\lambda_{\text{Het}} = \frac{1}{1 + \tau^{1-\sigma} \left(\frac{\varphi_d^T}{\varphi_x^T}\right)^{k-(\sigma-1)} \frac{1-\left(\frac{\varphi_x^T}{\varphi_{\max}}\right)^{k-(\sigma-1)}}{1-\left(\frac{\varphi_d^T}{\varphi_{\max}}\right)^{k-(\sigma-1)}}} > \lambda_{\text{Hom}} = \frac{1}{1 + \tau^{-(\sigma-1)}}. \tag{65}$$

Trade Elasticities: Given the same structural parameters ($f_d, f_e, f_x, \tau, L, \sigma$), the heterogeneous and homogeneous firm models also have different implications for the elasticity of trade flows with respect to trade costs. In the homogeneous firm model, the elasticity of trade with respect to trade costs is zero for parameter values for which the representative firm does not find it profitable to export. For parameter values for which the representative firm exports, there is a constant elasticity of trade with respect to variable trade costs and a zero elasticity of trade with respect to fixed trade costs:

$$\theta_{\text{Hom}}^{\tau} = -\frac{\mathrm{d}\ln\left(\frac{1-\lambda_{\text{Hom}}}{\lambda_{\text{Hom}}}\right)}{\mathrm{d}\ln\tau} = \begin{cases} (\sigma-1) & 0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 \\ 0 & 0 < 1 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} \end{cases}, \tag{66}$$

$$\theta_{\text{Hom}}^{f_x} = -\frac{\mathrm{d}\ln\left(\frac{1-\lambda_{\text{Hom}}}{\lambda_{\text{Hom}}}\right)}{\mathrm{d}\ln f_x} = \begin{cases} 0 & 0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 \\ 0 & 0 < 1 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} \end{cases}, \tag{67}$$

where trade increases discontinuously from zero to a positive value as trade costs fall below the value at which the representative firm begins to export: $\tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} = 1$.

In the heterogeneous firm model, the elasticities of trade with respect to variable and fixed trade costs are in general endogenous variables. For parameter values for which only some firms export ($\varphi_d^T < \varphi_x^T$), these endogenous variables depend on the functional form of the productivity distribution and the level of trade costs. For parameter values for which all firms export ($\varphi_d^T = \varphi_x^T$), the elasticities of trade with respect to variable and fixed trade costs are the same as in the homogeneous firm model:

$$\theta_{\text{Het}}^{\tau} = \begin{cases} (\sigma-1) - \frac{\mathrm{d}\ln\Lambda}{\mathrm{d}\ln\tau} > 0 & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1 \\ (\sigma-1) & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} < 1 \end{cases}, \tag{68}$$

$$\theta_{\text{Het}}^{f_x} = \begin{cases} -\frac{\mathrm{d}\ln\Lambda}{\mathrm{d}\ln f_x} > 0 & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1 \\ 0 & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} < 1 \end{cases}, \tag{69}$$

where the trade elasticity can change discontinuously as trade costs fall below the value at which only some firms export: $\tau\left(f_x/f_d\right)^{1/(\sigma-1)} = 1$.

Proposition 7 *Given the same structural parameters ($f_d, f_e,\ f_x,\ \tau,\ L,\ \sigma$), the elasticities of trade with respect to variable and fixed trade costs are strictly larger in absolute value in the heterogeneous firm model than in the homogeneous firm model for parameter values for which there is trade in both models and selection into export markets in the heterogeneous firm model ($0 < \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < 1 < \tau\left(f_x/f_d\right)^{1/(\sigma-1)}$). The trade elasticities are the same in the two models for parameter values for which all firms export in the heterogeneous firm model ($0 \leq \tau\left(f_x/\bar{F}_d\right)^{1/(\sigma-1)} < \tau\left(f_x/f_d\right)^{1/(\sigma-1)} \leq 1$).*

Proof. The proposition follows immediately from the trade elasticities in the homogeneous firm model (66) and (67) and the trade elasticities in the heterogeneous firm model (68) and (69), since $\frac{\mathrm{d}\Lambda}{\mathrm{d}\tau}\frac{\tau}{\Lambda} < 0$ and $\frac{\mathrm{d}\Lambda}{\mathrm{d}f_x}\frac{f_x}{\Lambda} < 0$. ■

Trade Elasticities with an Untruncated Pareto Productivity Distribution: In the special case of an untruncated Pareto distribution, the elasticities of trade with respect to variable and fixed trade costs are constants that depend only on whether or not there is selection into export markets, as can be seen from the domestic trade share (63):

$$\theta_{\mathrm{Het}}^{\tau} = \begin{cases} k & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1 \\ (\sigma-1) & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} < 1 \end{cases}, \tag{70}$$

$$\theta_{\mathrm{Het}}^{f_x} = \begin{cases} \frac{k-(\sigma-1)}{\sigma-1} & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} > 1 \\ 0 & \tau\left(f_x/f_d\right)^{1/(\sigma-1)} < 1 \end{cases}, \tag{71}$$

where the trade elasticity changes discontinuously as trade costs fall below the value at which only some firms export: $\tau\left(f_x/f_d\right)^{1/(\sigma-1)} = 1$.

Trade Elasticities with a Truncated Pareto Productivity Distribution: Even a slight generalization of an untruncated Pareto distribution to introduce a finite upper bound to the support of the productivity distribution ($\varphi_{\max} < \infty$) implies that the elasticities of trade with respect to variable and fixed trade costs become variable rather than constant. We now report the closed form solutions for these variable trade elasticities for a truncated Pareto productivity distribution. The elasticity of trade with respect to variable trade costs is:

$$\theta_{\mathrm{HetTR}}^{\tau} = -\frac{\mathrm{d}\ln\left(\frac{1-\lambda_{\mathrm{HetTR}}}{\lambda_{\mathrm{HetTR}}}\right)}{\mathrm{d}\ln\tau}.$$

Using the domestic trade share for a truncated Pareto distribution (65), the elasticity of trade with respect to variable trade costs can be expressed as:

$$\theta_{\mathrm{HetTR}}^{\tau} = (\sigma-1)+(k-(\sigma-1))\left[\frac{\left(\frac{\varphi_{\min}}{\varphi_x^T}\right)^{k-(\sigma-1)}\frac{\mathrm{d}\ln\varphi_x^T}{\mathrm{d}\ln\tau}}{\left(\frac{\varphi_{\min}}{\varphi_x^T}\right)^{k-(\sigma-1)}-\left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}} - \frac{\left(\frac{\varphi_{\min}}{\varphi_d^T}\right)^{k-(\sigma-1)}\frac{\mathrm{d}\ln\varphi_d^T}{\mathrm{d}\ln\tau}}{\left(\frac{\varphi_{\min}}{\varphi_d^T}\right)^{k-(\sigma-1)}-\left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}}\right] \tag{72}$$

4.1 ACR Welfare Derivation

No further derivations required.

4.2 Gains from Trade in the Homogeneous Firm Model

No further derivations required.

4.3 Gains from Trade in the Heterogeneous Firm Model

We now seek to express the welfare gains from trade liberalization in terms of observable empirical moments for the general case of our heterogeneous firm model. Since trade continuously drops to zero when trade costs increase, we can start from an open economy trade regime T without loss of generality. To simplify notation, we drop the T superscript. For now, we also assume that there is export market selection in this trade regime so that $\varphi_x > \varphi_d$.

Full and partial trade elasticities: From the domestic trade share (61), the full elasticity of trade with respect to variable trade costs (θ) can be expressed as follows::

$$\begin{aligned}\theta = -\frac{\mathrm{d}\ln\left(\frac{1-\lambda}{\lambda}\right)}{\mathrm{d}\ln\tau} &= (\sigma-1) - \frac{\mathrm{d}\ln\Lambda}{\mathrm{d}\ln\tau},\\ &= (\sigma-1) - \frac{\mathrm{d}\ln\delta(\varphi_x)}{\mathrm{d}\ln\tau} + \frac{\mathrm{d}\ln\delta(\varphi_d)}{\mathrm{d}\ln\tau},\\ &= (\sigma-1) - \frac{\mathrm{d}\ln\delta(\varphi_x)}{\mathrm{d}\ln\varphi_x}\frac{\mathrm{d}\ln\varphi_x}{\mathrm{d}\ln\tau} + \frac{\mathrm{d}\ln\delta(\varphi_d)}{\mathrm{d}\ln\varphi_d}\frac{\mathrm{d}\ln\varphi_d}{\mathrm{d}\ln\tau},\\ &= (\sigma-1) + \gamma(\varphi_x)\frac{\mathrm{d}\ln\varphi_x}{\mathrm{d}\ln\tau} - \gamma(\varphi_d)\frac{\mathrm{d}\ln\varphi_d}{\mathrm{d}\ln\tau}, \end{aligned} \tag{73}$$

where $\delta(\varphi_j) = \int_{\varphi_j}^{\varphi_{\max}} \varphi^{\sigma-1} dG(\varphi)$ is proportional to the cumulative market share (in any given market) of firms above any productivity cutoff φ_j; $\gamma(\varphi_j) = -\mathrm{d}\ln\delta(\varphi)/\mathrm{d}\ln\varphi_j$ is the elasticity of $\delta(\varphi_j)$ for market $j \in \{d, x\}$; hence $\gamma(\varphi_j)$ represents the hazard function for the distribution of log firm size within a market.

As argued by ACR, only the *partial* trade elasticity capturing the direct effect of τ is observed empirically, since it is estimated from a gravity equation with exporter and importer fixed effects. In the context of our symmetric country model, this partial elasticity can be derived from (61), which relates the domestic trade share to variable trade costs and the two productivity cutoffs ($\lambda = \lambda(\tau, \varphi_d, \varphi_x)$), and from (17), which relates the two productivity cutoffs to one another ($\varphi_x = \varphi_x(\tau, \varphi_d)$).[3] Taking the partial derivative of the domestic trade share with respect to τ holding φ_d constant, we have:

$$\vartheta = -\left.\frac{\partial\ln\left(\frac{1-\lambda}{\lambda}\right)}{\partial\ln\tau}\right|_{\varphi_d} = (\sigma-1) - \left.\frac{\partial\ln\Lambda}{\partial\ln\varphi_x}\,\frac{\partial\ln\varphi_x}{\partial\ln\tau}\right|_{\varphi_d},$$

where the relationship between the productivity cutoffs (17) implies $\partial\ln\varphi_x/\partial\ln\tau|_{\varphi_d} = 1$. Therefore the partial elasticity can be further written as:

$$\begin{aligned}\vartheta &= (\sigma-1) - \left.\frac{\partial\ln\Lambda}{\partial\ln\varphi_x}\right|_{\varphi_d},\\ &= (\sigma-1) + \gamma(\varphi_x),\end{aligned} \tag{74}$$

where $\gamma(\varphi_j) = -\mathrm{d}\ln\delta(\varphi)/\mathrm{d}\ln\varphi_j$ is the elasticity of $\delta(\varphi_j)$ for market $j \in \{d, x\}$.

Welfare with a general productivity distribution: Using the domestic trade share (61), welfare (24) can be re-expressed as:

$$\mathbb{W}_{\text{Het}} = \frac{\sigma-1}{\sigma} M_e^{\frac{1}{\sigma-1}} \left(\frac{\delta(\varphi_d)}{\lambda}\right)^{\frac{1}{\sigma-1}}, \tag{75}$$

where $\delta(\varphi_d)^{1/(\sigma-1)}$ is the productivity of a firm that has domestic market revenue equal to expected domestic market revenue per entering firm; the exponent on the domestic trade share ($-1/(\sigma-1)$)

[3] In the web appendix, we show how a multi-country version of our model yields an expression for changes in log bilateral trade that is linear in exporter and importer fixed effects and $\vartheta\ln\tau$.

depends on the elasticity of substitution. This relationship holds in both the closed economy ($\lambda = 1$) and an open economy equilibrium with selection into export markets ($\lambda < 1$).

Totally differentiating this expression for welfare (75), the change in welfare depends on the change in the mass of entrants, the change in the domestic trade share, and the change in expected domestic market productivity per entering firm:

$$\mathrm{d}\ln \mathbb{W}_{\mathrm{Het}} = \frac{1}{\sigma-1}\mathrm{d}\ln M_e - \frac{1}{\sigma-1}\mathrm{d}\ln\lambda + \frac{1}{\sigma-1}\mathrm{d}\ln\delta(\varphi_d). \tag{76}$$

Totally differentiating our earlier expression for welfare (26), the change in welfare is also equal to the change in the domestic productivity cutoff:

$$\mathrm{d}\ln \mathbb{W}_{\mathrm{Het}} = \mathrm{d}\ln\varphi_d. \tag{77}$$

Combining these two relationships and the definition of $\gamma(\varphi_d)$, the change in welfare following trade liberalization can be expressed in terms of the change in the domestic trade share, the change in the mass of entrants, a variable partial trade elasticity ϑ, and the difference in the hazard function $\gamma(\varphi_d) - \gamma(\varphi_x)$ between the domestic and export markets:

$$\mathrm{d}\ln \mathbb{W}_{\mathrm{Het}} = \frac{1}{\vartheta + [\gamma(\varphi_d) - \gamma(\varphi_x)]}\left[\mathrm{d}\ln M_e - \mathrm{d}\ln\lambda\right], \qquad \vartheta = \sigma - 1 + \gamma(\varphi_x), \tag{78}$$

Welfare with an untruncated Pareto productivity distribution: In the special case of an untruncated Pareto productivity distribution, the relationship between welfare and the domestic trade share can be simplified further using the following three results: (i) the mass of entrants is a constant that depends on parameters alone:

$$M_e = \frac{\sigma-1}{\sigma k}\frac{L}{f_e}; \tag{79}$$

(ii) expected productivity in each market per entering firm ($\delta_j^{1/(\sigma-1)}$ for market $j \in \{d, x\}$) is a constant elasticity function of the productivity cutoff for that market:

$$\delta_j = \frac{k}{k-(\sigma-1)}\varphi_{\min}^k\left(\varphi_j\right)^{-(k-(\sigma-1))}; \tag{80}$$

(iii) the trade share for each market is a constant elasticity function of the productivity cutoff for that market:

$$\lambda_j = \varphi_j^{-k}\left(\frac{M_e}{L}\right)\sigma f_d\left[\frac{k}{k-(\sigma-1)}\varphi_{\min}^k\right]. \tag{81}$$

Together these three results in turn imply: (a) a common constant hazard function in the domestic and export markets:

$$\gamma(\varphi_d) = \gamma(\varphi_x) = \gamma = k - (\sigma - 1); \tag{82}$$

(b) a constant partial trade elasticity:

$$\vartheta = \sigma - 1 + \gamma = k; \tag{83}$$

(c) a constant full trade elasticity that equals the partial trade elasticity:

$$\theta = \vartheta = k. \tag{84}$$

The property that the partial and full trade elasticities are both constant and equal to one another is specific to the untruncated Pareto distribution. In this special case, the domestic trade share (61) under export market selection ($\varphi_d^T < \varphi_x^T$) can be written as:

$$\lambda = \frac{1}{1 + \tau^{-(\sigma-1)}\left(\frac{\varphi_d^T}{\varphi_x^T}\right)^{k-(\sigma-1)}} = \frac{1}{1 + \tau^{-k}\left(\frac{f_x}{f_d}\right)^{-\frac{k-(\sigma-1)}{\sigma-1}}}, \tag{85}$$

and welfare (75) can be expressed solely in terms of the domestic trade share and parameters:

$$\mathbb{W}_{\text{Het}} = \lambda^{-\frac{1}{k}} L^{\frac{1}{\sigma-1}} \left[\frac{\varphi_{\min}^{k} f_d^{1-\frac{k}{\sigma-1}}}{f_e \left(\frac{\sigma}{\sigma-1}\right)^{k} \sigma^{\frac{k}{\sigma-1}}} \frac{\sigma-1}{k-(\sigma-1)} \right]^{\frac{1}{k}}, \tag{86}$$

where the exponent on the domestic trade share now depends on the Pareto shape parameter $(-1/k)$ rather than on the elasticity of substitution $(-1/(\sigma-1))$. Therefore changes in welfare depend solely on changes in the domestic trade share and a constant trade elasticity ($\theta = \vartheta = k$):

$$\mathrm{d}\ln \mathbb{W}_{\text{Het}} = -\frac{1}{k}\mathrm{d}\ln \lambda. \tag{87}$$

Even in this special case of an untruncated Pareto distribution, the trade elasticity remains a reduced-form object rather than a structural parameter. Instead of a single trade elasticity, there are two separate trade elasticities for variable and fixed trade costs. Therefore the elasticity of trade with respect to observed trade barriers can vary depending on the extent to which these changes in observed trade barriers affect fixed versus variable trade costs. Furthermore, the value of these trade elasticities depends on whether or not there is selection into export markets. As a result, changes to trade and production costs that move the economy between regions of the parameter space with and without export market selection lead to discrete changes in the trade elasticities (from k to $\sigma - 1$ for variable trade costs and from $(k-(\sigma-1))/(\sigma-1)$ to zero for fixed exporting costs). Hence, the use of a reduced-form trade elasticity in trade policy evaluation is subject to the Lucas Critique, because the trade elasticity estimated for one context need not apply in another context, and the trade elasticity is not invariant with respect to policy changes that move the economy between different regions of the parameter space. Restriction R3 in ACR abstracts from this discrete change in trade elasticities by restricting attention to the parts of the parameter space in the heterogeneous firm model with an untruncated Pareto distribution in which trade elasticities are constant.

Hsieh and Ossa (2011): A somewhat separate implication of he untruncated Pareto productivity distribution is for the *source* of the welfare gains from trade in the heterogeneous firm model. In general, trade liberalization affects the mass of varieties available for consumption through both the mass of varieties exported from foreign and the mass of domestically-produced varieties. In the special case of an untruncated Pareto distribution, these two effects exactly offset one another, so as to leave the trade-share-weighted proportional change in the mass of varieties available for consumption equal to zero, as shown by Feenstra (2010) and Hsieh and Ossa (2011). For a general continuous productivity distribution, these two effects no longer necessarily exactly offset one another, so that the trade-share-weighted proportional change in the mass of varieties available for consumption need not be zero.

In the remainder of this subsection, we report the derivations of the Hsieh and Ossa (2011) result for an untruncated Pareto distribution. We consider a single-sector version of Hsieh-Ossa and **use their notation for the remainder of this discussion**. Countries are indexed by i and j. Productivity is assumed to have the following Pareto distribution:

$$g_i(\varphi) = \theta b_i^{\theta} \varphi^{-(\theta+1)}. \tag{88}$$

The paper evaluates the effect of productivity growth (changes in b_i) holding constant trade costs and other parameters of the model. The zero profit productivity cutoff condition is:

$$\varphi_{ij}^{*} = \frac{\sigma}{\sigma-1}\frac{\tau_{ij} w_i}{P_j}\left(\frac{\sigma f_{ij}}{L_j}\right)^{\frac{1}{\sigma-1}}, \tag{89}$$

where fixed costs are incurred in the destination country and hence terms in w_j have cancelled from the final term in parentheses. Weighted average productivity is:

$$\widetilde{\varphi}_{ij} = \left(\frac{\theta}{\theta-\sigma+1}\right)^{\frac{1}{\sigma-1}} \varphi^*_{ij}. \tag{90}$$

The mass of firms exporting from i to j is:

$$M_{ij} = \left(\frac{b_i}{\varphi^*_{ij}}\right)^{\theta} M_{ei}. \tag{91}$$

The value of trade from i to j is:

$$T_{ij} = M_{ij}\left(\frac{\tau_{ij}w_i}{\tilde{\varphi}_{ij}}\right)^{1-\sigma}. \tag{92}$$

The price index in market j is:

$$P_j^{1-\sigma} = \sum_i M_{ij}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\tilde{\varphi}_{ij}}\right)^{1-\sigma}. \tag{93}$$

Totally differentiating the price index, we have:

$$\widehat{P}_j = \sum_i \frac{T_{ij}}{w_jL_j}\left(-\widehat{\widetilde{\varphi}}_{ij} + \widehat{w}_i - \frac{1}{\sigma-1}\widehat{M}_{ij}\right), \tag{94}$$

where, in the Hsieh-Ossa notation used in this subsection, a hat above a variable denotes a proportional change: $\hat{x} = \mathrm{d}x/x$. Totally differentiating the zero-profit productivity cutoff (89), we have:

$$\widehat{\varphi}^*_{ij} = \widehat{w}_i - \widehat{P}_j.$$

Totally differentiating weighted average productivity, we have:

$$\widehat{\widetilde{\varphi}}_{ij} = \widehat{w}_i - \widehat{P}_j,$$

which implies:

$$\sum_i \frac{T_{ij}}{w_jL_j}\widehat{\widetilde{\varphi}}_{ij} = \sum_i \frac{T_{ij}}{w_jL_j}\widehat{w}_i - \widehat{P}_j. \tag{95}$$

Totally differentiating the mass of exporters, we have:

$$\widehat{M}_{ij} = \theta\left(\widehat{b}_i - \widehat{\varphi}^*_{ij}\right) + \widehat{M}_{ei}.$$

Now use $\widehat{\varphi}^*_{ij} = \widehat{w}_i - \widehat{P}_j$ from above:

$$\widehat{M}_{ij} = \theta\left(\widehat{b}_i - \widehat{w}_i\right) + \theta\widehat{P}_j + \widehat{M}_{ei},$$

which implies:

$$\sum_i \frac{T_{ij}}{w_jL_j}\widehat{M}_{ij} = \theta\sum_i \frac{T_{ij}}{w_jL_j}\left(\widehat{b}_i - \widehat{w}_i\right) + \theta\widehat{P}_j + \sum_i \frac{T_{ij}}{w_jL_j}\widehat{M}_{ei}. \tag{96}$$

Now use this result in the proportional change in the price index (94):

$$\widehat{P}_j = \sum_i \frac{T_{ij}}{w_jL_j}\left(-\widehat{\widetilde{\varphi}}_{ij} + \widehat{w}_i\right) - \frac{\theta}{\sigma-1}\sum_i \frac{T_{ij}}{w_jL_j}\left(\widehat{b}_i - \widehat{w}_i\right) - \frac{\theta}{\sigma-1}\widehat{P}_j - \frac{1}{\sigma-1}\sum_i \frac{T_{ij}}{w_jL_j}\widehat{M}_{ei}.$$

Now use $\widehat{\widetilde{\varphi}}_{ij} = \widehat{w}_i - \widehat{P}_j$ from above. We get:

$$\widehat{P}_j = \sum_i \frac{T_{ij}}{w_j L_j} \left(-\widehat{b}_i + \widehat{w}_i - \frac{1}{\theta} \widehat{M}_{ei} \right). \tag{97}$$

Now use this expression for the proportional change in the price index in (95) to obtain:

$$\sum_i \frac{T_{ij}}{w_j L_j} \widehat{\widetilde{\varphi}}_{ij} = \sum_i \frac{T_{ij}}{w_j L_j} \widehat{b}_i + \frac{1}{\theta} \sum_i \frac{T_{ij}}{w_j L_j} \widehat{M}_{ei}. \tag{98}$$

Now re-arrange the proportional change in the price index (94):

$$\frac{1}{\sigma - 1} \sum_i \frac{T_{ij}}{w_j L_j} \widehat{M}_{ij} = \sum_i \frac{T_{ij}}{w_j L_j} \left(-\widehat{\widetilde{\varphi}}_{ij} + \widehat{w}_i \right) - \widehat{P}_j.$$

Now use the result (98) in the right-hand side to obtain:

$$\frac{1}{\sigma - 1} \sum_i \frac{T_{ij}}{w_j L_j} \widehat{M}_{ij} = -\sum_i \frac{T_{ij}}{w_j L_j} \widehat{b}_i - \frac{1}{\theta} \sum_i \frac{T_{ij}}{w_j L_j} \widehat{M}_{ei} + \sum_i \frac{T_{ij}}{w_j L_j} \widehat{w}_i - \widehat{P}_j.$$

Now notice that the first three terms on the right-hand side equal $\widehat{P}_j$ from (97). Therefore we have:

$$\sum_i \frac{T_{ij}}{w_j L_j} \widehat{M}_{ij} = 0,$$

which implies that the trade-share-weighted proportional change in the mass of varieties available for consumption is equal to zero. This concludes our discussion of Hsieh and Ossa (2011).

Heterogeneous Firm model with a Truncated Pareto Distribution: Even small departures from an untruncated Pareto distribution, such as the introduction of a finite upper bound to the support of the productivity distribution (a truncated Pareto distribution), imply that changes in welfare can be no longer summarized by changes in the domestic trade share and a constant trade elasticity.[4] In this more general case of a truncated Pareto distribution, the partial trade elasticity (ϑ) is variable, the hazard function differs between the domestic and export markets ($\gamma(\varphi_d) \neq \gamma(\varphi_x)$ for $\varphi_d \neq \varphi_x$), and the partial trade elasticity (ϑ) differs from the full trade elasticity (θ). The partial trade elasticity (ϑ), the hazard functions ($\gamma(\varphi_d)$ and $\gamma(\varphi_x)$) and the mass of entrants (M_e) all depend on the productivity cutoffs (and hence on trade costs). As a result, the welfare effects of trade liberalization must be determined using the general formulas (75) and (78) and no longer can be summarized by the domestic trade share and a constant trade elasticity in (86) and (87).

In this more general case of a truncated Pareto distribution, we again have a closed-form expression for the domestic trade share (61):

$$\lambda = \frac{1}{1 + \tau^{1-\sigma} \left(\frac{\varphi_d^T}{\varphi_x^T} \right)^{k-(\sigma-1)} \frac{1 - \left(\frac{\varphi_x^T}{\varphi_{\max}} \right)^{k-(\sigma-1)}}{1 - \left(\frac{\varphi_d^T}{\varphi_{\max}} \right)^{k-(\sigma-1)}}}, \tag{99}$$

[4] Helpman, Melitz and Rubinstein (2008) uses a truncated Pareto productivity distribution to rationalize zero bilateral trade flows in a gravity equation estimation. Feenstra (2014) uses a truncated Pareto distribution to generate welfare gains from trade through productivity selection, product variety and pro-competitive effects in a heterogeneous firm model with a quadratic mean of order r (QMOR) expenditure function.

where the untruncated Pareto distribution corresponds to the limiting case in which $\varphi_{\max} \to \infty$. In this more general case of a truncated Pareto distribution, the cumulative market share ($\delta(\varphi_j)$) for firms above a productivity cutoff (φ_j) in market $j \in \{d, x\}$ is:

$$\delta(\varphi_j) = \frac{k}{k-(\sigma-1)} \varphi_{\min}^{\sigma-1} \frac{\left(\frac{\varphi_{\min}}{\varphi_j}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}}{1-\left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k}}. \tag{100}$$

Hence the hazard function for the distribution of log firm size ($\gamma(\varphi_j)$) within market j is:

$$\gamma(\varphi_j) = (k-(\sigma-1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_j^T}\right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_j^T}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}}. \tag{101}$$

As $\varphi_{\max} \to \infty$, the hazard function $\gamma(\varphi_j)$ converges to its constant value for an untruncated Pareto distribution: $\lim_{\varphi_{\max}\to\infty} \gamma(\varphi_j) = k-(\sigma-1)$. More generally, for $\varphi_{\max} < \infty$, $\gamma(\varphi_j)$ takes a strictly higher value than for an untruncated Pareto productivity distribution and differs between the domestic and export market. The hazard function for each market is increasing in the productivity cutoff, attaining its minimum value as $\varphi_j \to \varphi_{\min}$, and converging towards infinity as $\varphi_j \to \varphi_{\max}$. Since higher variable trade costs reduce the domestic productivity cutoff and increase the export productivity cutoff, they imply a *lower* $\gamma(\varphi_d)$ and a *higher* $\gamma(\varphi_x)$.

In this more general case of a truncated Pareto distribution, changes in the domestic trade share ($\mathrm{d}\ln\lambda$) and a constant trade elasticity ($\vartheta = \theta = k$) are no longer sufficient to determine the welfare effects of trade liberalization. Instead, the partial trade elasticity (ϑ) is variable, and the effects of trade liberalization on welfare also depend on the difference in hazard functions between the domestic and export markets ($\gamma(\varphi_d) \neq \gamma(\varphi_x)$) and the change in the mass of entrants ($\mathrm{d}\ln M_e$). In Section 6 of the paper, we compare the true welfare effects of trade liberalization for a truncated Pareto distribution (using (75) and (78)) to the welfare effects that a researcher would predict if they falsely assumed a constant trade elasticity (using (86) and (87)). We show that assuming a constant trade elasticity when the partial trade elasticity is variable and the hazard function differs across markets can lead to substantial discrepancies between the predicted and true welfare effects of trade liberalization.

Asymmetric Countries: We now consider a world of many asymmetric countries. We show that the partial trade elasticity corresponds to the trade elasticity for bilateral trade net of an exporter and importer fixed effect. We generalize our analysis of welfare in the heterogeneous firm model with a general productivity distribution to this case of many asymmetric countries. We use the first subscript to denote the exporter (typically i) and the second subscript to denote the importer (typically j). Using this notation, bilateral exports from i to j can be written as follows:

$$R_{ij} = M_{ij}\bar{r}_{ij},$$

$$R_{ij} = M_{ij}\left[\int_{\varphi_{ij}}^{\varphi_{\max}} \left(\frac{\varphi}{\varphi_{ij}}\right)^{\sigma-1} \frac{\mathrm{d}G(\varphi)}{1-G(\varphi_{ij})}\right]\sigma f_{ij},$$

$$R_{ij} = [1-G(\varphi_{ij})]\, M_{ei}\left[\int_{\varphi_{ij}}^{\varphi_{\max}} \left(\frac{\varphi}{\varphi_{ij}}\right)^{\sigma-1} \frac{\mathrm{d}G(\varphi)}{1-G(\varphi_{ij})}\right]\sigma f_{ij},$$

$$R_{ij} = M_{ei}\delta(\varphi_{ij})\varphi_{ij}^{-(\sigma-1)}\sigma f_{ij}.$$

We assume that bilateral exporting costs (f_{ij}) can be partitioned into a component that depends on exporter characteristics (f_i) and a component that depends on importer characteristics (f_j). Using this assumption, bilateral exports become:

$$R_{ij} = M_{ei}\delta(\varphi_{ij})\varphi_{ij}^{-(\sigma-1)}\sigma f_j f_i,$$

which can be written as:

$$\ln R_{ij} = n_i + \xi_j + \ln \delta(\varphi_{ij}) - (\sigma - 1)\ln \varphi_{ij},$$

where η_i is an exporter fixed effect; ξ_j is an importer fixed effect; and we have absorbed σ into the definitions of the fixed effects. Therefore we have:

$$\left.\frac{\partial \ln R_{ij}}{\partial \ln \tau_{ij}}\right|_{\eta_i,\xi_j} = \frac{\mathrm{d}\ln \delta(\varphi_{ij})}{\mathrm{d}\ln \varphi_{ij}} \left.\frac{\partial \varphi_{ij}}{\partial \tau_{ij}}\right|_{\eta_i,\xi_j} - (\sigma - 1) \left.\frac{\partial \ln \varphi_{ij}}{\partial \ln \tau_{ij}}\right|_{\eta_i,\xi_j}$$

Now define:

$$\gamma(\varphi_{ij}) = -\frac{\mathrm{d}\ln \delta(\varphi_{ij})}{\mathrm{d}\ln \varphi_{ij}}.$$

and note that the exporting productivity cutoff condition (16) implies:

$$\left.\frac{\partial \ln \varphi_{ij}}{\partial \ln \tau_{ij}}\right|_{\eta_i,\xi_j} = 1.$$

We therefore have:

$$\left.\frac{\partial \ln R_{ij}}{\partial \ln \tau_{ij}}\right|_{\eta_i,\xi_j} = -\gamma(\varphi_{ij}) - (\sigma - 1)\,,$$

which can be written as:

$$-\left.\frac{\partial \ln R_{ij}}{\partial \ln \tau_{ij}}\right|_{\eta_i,\xi_j} = \vartheta_{ij}, \qquad \vartheta_{ij} = (\sigma - 1) + \gamma(\varphi_{ij}).$$

In the special case of an untruncated Pareto productivity distribution, we have:

$$\vartheta_{ij} = \vartheta = k, \qquad \text{for all } i, j. \tag{102}$$

Therefore, in this special case, there is a single constant partial trade elasticity for all pairs of source and destination countries. Assuming that variable trade costs are observed, and fixed trade costs are either also observed or can be captured by exporter and importer fixed effects, this single constant partial trade elasticity can be estimated as the coefficient on variable trade costs from a gravity equation including exporter and importer fixed effects.

In contrast, for a general productivity distribution, there is no single partial trade elasticity for all pairs of source and destination countries. Instead $\gamma(\varphi_{ij})$ is a variable that depends on the productivity cutoff (φ_{ij}) for each pair of source and destination countries and hence on the level of variable trade costs (τ_{ij}). We provide the closed form solution for $\gamma(\varphi_{ij})$ for a truncated Pareto productivity distribution in (101) above. Since the partial trade elasticity is a variable, the coefficient on variable trade costs from a gravity equation including exporter and importer fixed effects captures the average value of this elasticity across exporter-importer pairs for the regression sample. This average elasticity need not provide a good approximation to the partial trade elasticity for any one individual exporter-importer pair within or outside the regression sample.

We now show that our expression for the proportional change in welfare with two symmetric countries generalizes to the case of many asymmetric countries. The price index for country j is:

$$P_j = \frac{\sigma}{\sigma - 1}\left[\sum_i M_{ij}\left(\tau_{ij} w_i\right)^{1-\sigma}\left(\tilde{\varphi}_{ij}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}. \tag{103}$$

The share of importer j's expenditure on exporter i is:

$$\lambda_{ij} = \frac{M_{ij} \left(\tau_{ij} w_i\right)^{1-\sigma} \left(\tilde{\varphi}_{ij}\right)^{\sigma-1}}{\sum_k M_{kj} \left(\tau_{kj} w_k\right)^{1-\sigma} \left(\tilde{\varphi}_{kj}\right)^{\sigma-1}}. \tag{104}$$

Using the price index (103) and domestic trade share (104), welfare can be written as:

$$\mathbb{W}_j = \frac{w_j}{P_j} = \frac{\sigma-1}{\sigma} \left[M_{jj} \left(\tau_{jj} w_j\right)^{1-\sigma} \left(\tilde{\varphi}_{jj}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{105}$$

Noting that $\tau_{jj} = 1$, $M_{jj} = \left[1 - G(\varphi_{jj})\right] M_{ej}$, and using the definition of $\delta(\varphi_{jj})$, welfare can be written as:

$$\mathbb{W}_j = \frac{\sigma-1}{\sigma} M_{ej}^{\frac{1}{\sigma-1}} \left(\frac{\delta\left(\varphi_{jj}\right)}{\lambda_{jj}}\right)^{\frac{1}{\sigma-1}}. \tag{106}$$

Therefore the proportional change in welfare can be written as:

$$\mathrm{d}\mathbb{W}_j = \frac{1}{\vartheta_{jj}} \left[\mathrm{d}\ln M_{ej} - \mathrm{d}\ln \lambda_{jj}\right], \qquad \vartheta_{jj} = \sigma - 1 + \gamma(\varphi_{jj}), \tag{107}$$

where ϑ_{jj} is the domestic partial trade elasticity. Equivalently, this proportional change in welfare can be re-written as:

$$\mathrm{d}\mathbb{W}_j = \frac{1}{\vartheta_{ji} + \left[\gamma(\varphi_{jj}) - \gamma(\varphi_{ji})\right]} \left[\mathrm{d}\ln M_{ej} - \mathrm{d}\ln \lambda_{jj}\right], \tag{108}$$

where ϑ_{ji} is the partial trade elasticity for exporter j and importer i; $\gamma(\varphi_{jj}) - \gamma(\varphi_{ji})$ captures the hazard differential between country j's domestic market and its export market i.

5 Trade Policy Evaluation

No further derivations required.

6 Quantitative Relevance

In this section, we examine the quantitative relevance of our results. In Subsection 6.1, we show that our theoretical comparative static is associated with quantitatively relevant differences in welfare between the heterogeneous and homogeneous firm models. In Subsection 6.2, we show that the reduced-form nature of the trade elasticity is consequential for the practical evaluation of trade policies.

6.1 Theoretical Comparative Static

In this subsection, we compare the welfare properties of the heterogeneous and homogeneous firm models holding all structural parameters other than the productivity distribution constant between the two models and assuming an untruncated Pareto distribution in the heterogeneous firm model. We choose standard values for the model's parameters based on central estimates from the existing empirical literature and moments of the U.S. data.

We set the elasticity of substitution between varieties $\sigma = 4$, which is consistent with the estimates using plant-level U.S. manufacturing data in Bernard, Eaton, Jensen and Kortum (2003). The Pareto shape parameter for the productivity distribution (k) determines the elasticity of trade flows with respect to variable trade costs in the heterogeneous firm model under export market selection. We set $k = 4.25$ as a central value for estimates of the trade elasticity.[5] A choice for the Pareto scale parameter is equivalent to a choice of units in which to measure productivity, and hence we normalize $\varphi_{\min} = 1$.

We consider trade between two symmetric countries, and choose labor in one country as the numeraire ($w = 1$), which implies that the wage in both countries is equal to one. The general equilibrium of the model under the assumption of an untruncated Pareto distribution can be summarized by the following system of equations:

$$\varphi_d^T = \left[\frac{\sigma - 1}{k - (\sigma - 1)} \left(\frac{f_d + \tau^{-k} \left(\frac{f_x}{f_d} \right)^{\frac{-k}{\sigma - 1}} f_x}{f_e} \right) \varphi_{\min}^k \right]^{\frac{1}{k}}.$$

$$\begin{aligned}
\varphi_x^T &= \tau \left(\frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}} \varphi_d^T, \\
M_e &= \frac{L}{f_e} \frac{\sigma - 1}{\sigma k}, \\
M &= \left(\frac{\varphi_{\min}}{\varphi_d^T} \right)^k M_e, \\
R &= L, \\
\bar{r} &= \frac{f_e}{1 - G\left(\varphi_d^T\right)} \frac{\sigma k}{\sigma - 1}.
\end{aligned}$$

Inspecting this system of equations, it is clear that scaling L and $\{f_e,\ f_d,\ f_x\}$ up or down by the same proportion leaves the productivity cutoffs $\{\varphi_d,\ \varphi_x\}$ and the mass of entrants unchanged (M_e), and merely scales average firm size ($\bar{r}$) up or down by the same proportion. Therefore we set L equal to the U.S. labor force and normalize f_d to one. With an untruncated Pareto productivity distribution, the sunk entry cost f_e affects the absolute levels of the productivity cutoffs and welfare but not their relative levels for different values of trade costs. As a result, the relative comparisons below are invariant to the choice of f_e, and hence we normalize f_e to one.

We calibrate τ to match the average fraction of exports in firm sales in U.S. manufacturing ($\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} = 0.14$, as reported in Bernard, Jensen, Redding and Schott 2007), which implies $\tau = 1.83$ (which is in line with the estimate of 1.7 in Anderson and van Wincoop 2004). Given our choice for the parameters $\{\sigma,\ k,\ \varphi_{\min},\ f_d,\ f_e,\ \tau\}$, we choose f_x to ensure that the model is consistent with the average fraction of U.S. manufacturing firms that export (0.18, as reported in Bernard, Jensen, Redding and Schott 2007).

We choose the degenerate productivity distribution in the homogeneous firm model so that the two models generate the same aggregate variables in an initial equilibrium. In the main paper, we do so for an initial open economy equilibrium using our calibrated values of trade costs of $\tau = 1.83$ and $f_x = 0.545$. Therefore we compare the heterogeneous firm model to the extended homogeneous firm model introduced in subsection 3.2.

In this web appendix, we undertake the analysis for an initial closed economy equilibrium, as analyzed in subsection 3.1. We solve for the closed economy equilibrium of the heterogeneous firm model, including the probability of successful firm entry $\left[1 - G\left(\varphi_d^A\right)\right]$ and weighted average productivity ($\tilde{\varphi}_d^A$). In the homogeneous firm model, we set the probability of successful firm entry and productivity conditional on successful entry equal to these values: $\left[1 - \bar{G}_d\right] = \left[1 - G\left(\varphi_d^A\right)\right]$ and $\bar{\varphi}_d = \tilde{\varphi}_d^A$. All

[5] Simonovska and Waugh (2014a) estimate a trade elasticity of 4.10 or 4.27 depending on the data used. Costinot and Rodriguez-Clare (2013)'s benchmark value for the trade elasticity is 5. Any of these values would lead to quantitatively similar results.

parameters besides the productivity distribution $\{f_d,\ f_e,\ f_x,\ \tau,\ L,\ \sigma\}$ are assumed to be the same in the two models, which implies $\bar{F}_d = f_d + f_e / \left[1 - G\left(\varphi_d^A\right)\right]$. Given these same parameters, welfare and all other aggregate variables are identical in the closed economy equilibria of the two models.

We examine the effect of reducing trade costs from their infinite values under autarky to a range of finite values in the open economy equilibrium. In Figure A.1, we set fixed exporting costs equal to their calibrated value of ($f_x = 0.545$) and consider reductions in variable trade costs from infinity to $\tau \in [1, 2.5]$ (including the calibrated value of 1.83).[6] Panel A displays the welfare gains from opening the closed economy to trade ($\mathbb{W}^T/\mathbb{W}^A$); Panel B displays the probability of exporting (χ); Panel C displays domestic weighted average productivity relative to its value under autarky ($\tilde{\varphi}_d^T/\tilde{\varphi}_d^A$); Panel D displays the domestic trade share (λ). The solid blue line shows values in the heterogeneous firm model, while the red dashed line shows values in the homogeneous firm model.

As shown in Panel A, welfare in the open economy equilibrium is strictly greater in the heterogeneous firm model than in the homogeneous firm model for all finite values of variable trade costs. Across the range of variable trade costs shown in the figure, the welfare gains from trade range from 1.00 to 1.21, which is broadly in line with existing empirical estimates from quantitative trade models.

For sufficiently high variable trade costs, the representative firm does not find it profitable to export in the homogeneous firm model. At this value for variable trade costs, the probability of exporting in the homogeneous firm model falls from one to zero (Panel B); the domestic trade share in the homogeneous firm model rises to one (Panel D); and welfare in the homogeneous firm model is equal to its autarkic value, even though there remain substantial welfare gains from trade in the heterogeneous firm model (Panel A).

However, even for variable trade costs for which there is trade in both models, the heterogeneous firm model generates substantially higher welfare than in the homogeneous firm model. For example, for $\tau = 1.60$, the difference in welfare gains from trade between the two models (two percentage points) is as large as the overall welfare gains from trade in the homogeneous firm model (two percentage points). These differences in the welfare gains from trade between the two models are driven by the endogenous responses of the domestic and export productivity cutoffs to changes in trade costs in the heterogeneous firm model. In Panel C, productivity is constant by assumption in the homogeneous firm model. In contrast, weighted average productivity in the domestic market in the heterogeneous firm model rises by around 10 percent relative to its autarkic value as variable trade costs fall to one.

For sufficiently low values of variable trade costs, all firms export in the heterogeneous firm model. For this range of parameter values, the probability of exporting is one in both models (Panel B); the domestic trade share is the same in the two models (Panel D); and once all firms export further reductions in variable trade costs leave weighted average productivity unchanged (Panel C). Even for this range of trade costs (including $\tau = 1$), welfare in the heterogeneous firm model is strictly higher than in the homogeneous firm model, because of positive fixed exporting costs, which imply that the domestic productivity cutoff is different in the open and closed economy of the heterogeneous firm model.

Taken together, these results suggest that for empirically plausible parameter values the differences in the aggregate welfare predictions of the heterogeneous and homogeneous firm models are of quantitative relevance.

6.2 Practical Evaluation of Trade Policies

In the paper, we calibrate the upper bound to the support of the truncated Pareto productivity distribution ($\varphi_{\max} = 2.85$) using data on average size differences between exporters and non-exporters. We derive a closed-form solution for the hazard function for a truncated Pareto distribution (equation (31) in the paper and (101) in this web appendix). From this closed-form solution, our result that the

[6]For brevity, we concentrate on changes in variable trade costs, but find a similar pattern of results for changes in fixed exporting costs, as reported in the working paper version of the paper.

partial trade elasticity becomes large for sufficiently high variable trade costs is robust to alternative choices for the upper bound to the support of the productivity distribution ($\varphi_{\max}$), because there exists a sufficiently high trade cost such that φ_x^T converges to any finite value of $\varphi_{\max}$.

In this subsection, we illustrate this robustness of our results using an alternative choice for the upper bound to the support of the truncated Pareto productivity distribution of ($\varphi_{\max} = 4$). In Figure A.2, we examine each of the components of the proportional change in welfare (78) for this truncated Pareto productivity distribution. Panel A shows the partial trade elasticity (ϑ); Panel B displays the hazard differential between the domestic and export markets ($\gamma(\varphi_d) - \gamma(\varphi_x)$); Panel C shows the domestic trade share (λ); Panel D displays the mass of entrants (M_e). We change variable trade costs from their calibrated value of $\tau^{T_0} = 1.83$ to values of $\tau^{T_1} \in [1, 4]$ for which trade occurs.

In the special case in which the upper bound to the support of the productivity distribution converges to infinity ($\varphi_{\max} \to \infty$), the truncated Pareto distribution converges to an untruncated Pareto distribution. In this special case, the partial trade elasticity (ϑ) is constant and equal to the full trade elasticity (θ), which is equal to the Pareto shape parameter ($k = 4.25$). Furthermore, in this special case, the mass of entrants depends only on parameters and hence is constant.

In contrast, for a truncated Pareto distribution with a finite upper bound to the support of the productivity distribution ($\varphi_{\max} < \infty$), the partial trade elasticity (ϑ) is variable and differs from both the full trade elasticity (θ) and the Pareto shape parameter ($k = 4.25$). As we vary variable trade costs from one to four, the partial trade elasticity in Panel A ranges from three to more than fifteen.[7] As variable trade costs increase, the export productivity cutoff (φ_x^T) rises, which increases the export hazard ($\gamma(\varphi_x^T)$) and hence in turn increases the partial trade elasticity (ϑ). As variable trade costs become sufficiently large that the export productivity cutoff approaches the upper bound to the support of the productivity distribution ($\varphi_x \to \varphi_{\max}$), the partial trade elasticity converges towards infinity ($\vartheta \to \infty$). As variable trade costs become sufficiently small that all firms export, the export and domestic productivity cutoffs become equal to one another ($\varphi_x^T = \varphi_d^T$) and independent of variable trade costs. At the threshold value for variable trade costs below which all firms export, the partial trade elasticity (ϑ) falls discretely to $\sigma - 1$ and remains equal to this constant value for all lower variable trade costs. Taken together, these results suggest that the partial trade elasticity can vary quite substantially from one context to another.

As shown in Panel B, these changes in variable trade costs have implications for the difference in hazard functions between the domestic and export markets ($\gamma(\varphi_d^T) - \gamma(\varphi_x^T)$). As variable trade costs increase, the resulting rise in the export productivity cutoff (φ_x^T) *increases* the hazard function in the export market ($\gamma(\varphi_x^T)$), but the associated reduction in the domestic productivity cutoff (φ_d^T) *reduces* the hazard function in the domestic market ($\gamma(\varphi_d^T)$). As a result, as we vary variable trade costs from one to four, the hazard rate differential between the two markets ranges from zero to minus twelve. As variable trade costs become sufficiently large that the export productivity cutoff approaches the upper bound to the support of the productivity distribution ($\varphi_x \to \varphi_{\max}$), the difference between the two hazard functions converges towards minus infinity ($\gamma(\varphi_d^T) - \gamma(\varphi_x^T) \to -\infty$). As variable trade costs become sufficiently small that all firm export, the export and domestic productivity cutoffs become equal to one another ($\varphi_x^T = \varphi_d^T$), and the difference between the two hazard functions becomes equal to zero ($\gamma(\varphi_d^T) - \gamma(\varphi_x^T) = 0$). These results highlight the importance of controlling for differences across markets in the hazard function of the distribution of log firm size when computing the welfare gains from trade liberalization using (78).

As shown in Panel C, increases in variable trade costs raise the domestic trade share, which converges towards one as variable trade costs rise towards four, and converges towards a value of one half as variable trade costs fall towards one (reflecting country symmetry). As shown in Panel D, increases in variable trade costs raise the mass of entrants, which is shown for each value of variable trade costs in the figure relative to its value for $\tau = 1$. With a truncated Pareto distribution, higher

[7] Under the assumption of a constant elasticity of trade costs with respect to distance, Novy (2013) estimates elasticities of trade with respect to trade costs that range from less than five to more than twenty.

variable trade costs reduce average firm size conditional on successful entry. With a fixed labor endowment, this in turn leads to a larger mass of entrants. For the parameterization considered here, these changes in the mass of entrants are relatively small, with the mass of entrants increasing by less than 2 percent as variable trade costs increase from one to four. As variable trade costs become sufficiently small that all firms export, the export and domestic productivity cutoffs become equal to one another ($\varphi_x^T = \varphi_d^T$) and independent of variable trade costs. Therefore, for this range of variable trade costs, both average firm size and the mass of entrants are constant.

We now examine the quantitative implications of the above changes in micro structure for the evaluation of trade policies. Table A.1 compares the true welfare gains from trade liberalization with a truncated Pareto distribution to the welfare gains that would be predicted by a policy analyst who falsely assumed a constant trade elasticity and applied the ACR formula. We examine trade liberalization from high variable trade costs for which the economy is relatively closed ($\tau = 4$ and $\lambda = 0.999$), through intermediate values of variable trade costs ($\tau = 1.5$ and $\lambda = 0.826$), and to low values of variable trade costs for which the economy is relatively open but still only some firms export ($\tau = 1.25$ and $\lambda = 0.669$).

In Column (1), we report the true relative change in welfare ($\mathbb{W}^{T_1}/\mathbb{W}^{T_0}$) in the heterogeneous firm model with a truncated Pareto distribution (as computed using (75)). Reducing variable trade costs from $\tau = 4$ to $\tau = 1.25$ increases welfare by 8.72 percent, which is broadly in line with estimates of the welfare gains from trade in recent quantitative trade models. Around half of these welfare gains are achieved from the reduction in variable trade costs from $\tau = 4$ to $\tau = 1.5$ (3.92 percent), with the remaining half realized from a further reduction in variable trade costs to $\tau = 1.25$ (4.61 percent). Since the variable partial trade elasticity is increasing in variable trade costs, larger welfare gains are generated from a given percentage reduction in variable trade costs when the economy is relatively open than when it is relatively closed. This property of a variable trade elasticity has important implications for the evaluation of future efforts at multilateral trade liberalization. Even if variable trade costs already have been reduced to relatively low levels, this does not necessarily mean that most of the welfare gains from reductions in variable trade costs already have been achieved.

In Column (2), we report the results of an *ex ante* policy evaluation under the (false) assumption of a constant trade elasticity. We consider a policy analyst who has access to estimates of the partial trade elasticity for an initial value of trade costs (ϑ^{start}). The policy analyst considers each of the reductions in variable trade costs (e.g. from $\tau = 4$ to $\tau = 1.5$) and uses the ACR formula to predict the welfare effects of these trade liberalizations based on the observed change in the domestic trade share and the assumption of a constant trade elasticity.

For trade liberalizations starting from high variable trade costs (the first and second rows), we find substantial discrepancies between the true and predicted welfare gains from trade liberalization. Reducing variable trade costs from $\tau = 4$ to $\tau = 1.25$ is predicted in Column (2) to increase welfare by 2.44 percent (a discrepancy of more than six percentage points or more than 70 percent). These discrepancies arise because the true trade elasticity is variable rather than constant and because the hazard function differs between the domestic and export markets. For high values of variable trade

	(1)	(2)	(3)	(4)
Trade Liberalization	Actual (Truncated Pareto)	Predicted (ACR formula) ϑ^{start}	Predicted (ACR formula) $\vartheta^{\text{average}}$	Predicted (ACR formula) $\theta^{\text{start}}_{\text{end}}$
$\tau = 4$ to $\tau = 1.25$	108.72%	102.44%	106.15%	106.80%
$\tau = 4$ to $\tau = 1.5$	103.92%	101.15%	102.80%	103.05%
$\tau = 1.5$ to $\tau = 1.25$	104.61%	104.54%	104.59%	104.61%

Table A.1: Actual and Predicted Welfare Gains from Trade Liberalization costs, the partial trade elasticity changes substantially across different values of trade costs (Panel A of Figure A.2) and the difference in the hazard function between the domestic and export market is large (Panel B of Figure A.2). In contrast, for reductions in variable trade costs from intermediate to low values (the third row), we find that the predicted and true welfare effects of trade liberalization are relatively close to one another (a discrepancy of less than one percentage point). At these lower values of variable trade costs, the partial trade elasticity is relatively stable (Panel A of Figure A.2), and the difference in the hazard function between the domestic and export markets is small (Panel B of Figure A.2), because the export and domestic productivity cutoffs are close together.

In Columns (3) and (4), we report the results of an *ex post* policy evaluation under the (false) assumption of a constant trade elasticity. We consider a policy analyst who has access to an estimate of the average trade elasticity in between the start and end values of variable trade costs. The policy analyst considers each of the reductions in variable trade costs (e.g. from $\tau = 4$ to $\tau = 1.5$) and uses the ACR formula to predict the welfare effects of these trade liberalizations based on the observed change in the domestic trade share and the estimated average trade elasticity. We consider two different estimates for the average trade elasticity. In Column (3), we compute an average partial trade elasticity by considering variable trade costs at intervals of 0.005, evaluating the partial trade elasticity at each of these points, and taking the arithmetic mean of the partial trade elasticities across these points ($\vartheta^{\text{average}}$). In Column (4), we compute an average full trade elasticity by evaluating the logarithmic percentage growth in trade between the start and end values of trade costs and dividing this by the logarithmic percentage reduction in variable trade costs ($\theta_{\text{end}}^{\text{start}}$). Although the estimated average trade elasticity in Column (4) is a full elasticity rather than a partial elasticity, in practice we find similar results in both Columns (3) and (4).

For trade liberalization starting from high variable trade costs (the first and second rows), we continue to find quantitatively relevant discrepancies between the true and predicted welfare gains from trade liberalization. Reducing variable trade costs from $\tau = 4$ to $\tau = 1.25$ is predicted in Column (3) to increase welfare by 6.15 percent (a discrepancy of over 2.5 percentage points or around 29 percent of the true welfare gain from trade liberalization). In contrast, for reductions in variable trade costs from intermediate to low values (the third row), we find that the predicted and true welfare effects of trade liberalization are relatively close to one another (a discrepancy of less than one percentage point). Again this reflects the relative stability of the partial trade elasticity (Panel A of Figure A.2) and the small difference between the domestic and export hazards (Panel B of Figure A.2) at low values of variable trade costs. Unsurprisingly, the difference between the true and predicted welfare effects of trade liberalization is smaller using an average estimated trade elasticity in an *ex post* evaluation than using an initial estimated trade elasticity in an *ex ante* evaluation.

Key takeaways from this section are that both the partial trade elasticity and the hazard differential between the domestic and export markets can vary substantially across different values for variable trade costs (and hence in a multi-country world across relatively open and relatively closed economies). Taking a trade elasticity estimated from a relatively closed economy and applying this elasticity to a relatively open economy without controlling for the difference in hazard functions between the two markets can lead to quantitatively relevant discrepancies between the predicted and true welfare effects of trade liberalization in both *ex ante* and *ex post* evaluations. In contrast, taking a trade elasticity estimated from a relatively open economy and applying it to another relatively open economy provides a much closer approximation to the true welfare effects of trade liberalization.

We focus our quantitative analysis in this subsection on the truncated Pareto distribution to highlight that only a small departure in assumptions from an untruncated Pareto distribution can induce substantial variation in partial trade elasticities and substantial differences in the hazard function between the domestic and export markets. But the point that the partial trade elasticity is variable and the hazard function differs across markets is much more general, and also applies for example with a log normal distribution, as examined in Head, Mayer and Thoenig (2014).

7 Conclusions

No further derivations required.

8 Revealed Preference

The theoretical results throughout the paper are proved using the free entry condition in the market equilibrium of the heterogeneous firm model. But to provide further economic intuition for these results, we consider the problem of a social planner choosing the productivity cutoffs and the mass of entrants to maximize welfare in the heterogeneous firm model. We begin with the planner's problem in the closed economy. We next consider the planner's problem in the open economy. The planner is assumed to maximize world welfare, which with symmetric countries is equivalent to maximizing the welfare of the representative consumer in each country.[8] We show that the social planner's choices in the closed and open economies coincide with the market allocations, and hence the market allocations in the heterogeneous firm model are efficient.[9] We also show that the social planner in general chooses to adjust the productivity cutoffs following the opening of trade, even though it is feasible to leave them unchanged and replicate the open economy equilibrium of the homogeneous firm model. Therefore, by revealed preference, open economy welfare must be at least as high in the heterogeneous firm model as in the homogeneous firm model, and we show that it is in general higher.

8.1 Closed Economy

The real consumption index for the representative consumer is:

$$Q = \left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} q(\varphi)^{(\sigma-1)/\sigma} \, \mathrm{d}G(\varphi) \right]^{\sigma/(\sigma-1)}. \tag{109}$$

The social planner chooses the productivity cutoff φ_d^A, the output levels $q(\varphi)$ for all producing firms $\varphi \geq \varphi_d^A$, and the mass of entrants M_e to maximize Q subject to the aggregate labor constraint:

$$L = M_e \left\{ \int_{\varphi_d^A}^{\varphi_{\max}} \frac{q(\varphi)}{\varphi} \mathrm{d}G(\varphi) + \left[1 - G(\varphi_d^A)\right] f_d + f_e \right\}, \tag{110}$$

where the social planner faces the same productivity distribution $G(\varphi)$ and entry cost f_e per firm as in the market allocation.

The planner chooses the output levels $q(\varphi)$ to equate the marginal rates of transformations and marginal rates of substitution for firms with different productivities. The marginal rate of substitution between varieties for any two firms with productivities φ_1 and φ_2 is:

$$MRS = \left(\frac{q(\varphi_1)}{q(\varphi_2)} \right)^{1/\sigma}.$$

The marginal rate of transformation between varieties for any two firms with productivities φ_1 and φ_2 is:

$$MRT = \frac{\varphi_1}{\varphi_2}.$$

[8] To highlight the efficiency properties of the market equilibrium, we assume a world planner, which abstracts from the incentives of national planners to manipulate the terms of trade between countries.

[9] For an analysis of how the efficiency of the monopolistically competitive equilibrium depends on the extent to which the elasticity of substitution between varieties is constant or variable, see Dixit and Stiglitz (1977) for homogeneous firm models and Dhingra and Morrow (2012) for heterogeneous firm models.

Efficiency requires:

$$MRS = MRT \iff \frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

which yields the same relationship between relative quantities and relative productivities as in the market equilibrium. Using this relationship, we can rewrite the consumption index Q and the aggregate labor constraint as a function of the output level $\tilde{q}_d^A \equiv q(\tilde{\varphi}_d^A)$ of a firm with a weighted average productivity $\tilde{\varphi}_d^A$:

$$\begin{aligned} Q &= \left\{\left[1 - G(\varphi_d^A)\right] M_e\right\}^{\sigma/(\sigma-1)} \tilde{q}_d^A, \\ L &= \left[1 - G(\varphi_d^A)\right] M_e \left[\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} + f_d + \frac{f_e}{1 - G(\varphi_d^A)}\right], \end{aligned}$$

where we have used $q(\varphi) = \left(\varphi/\tilde{\varphi}_d^A\right)^{\sigma} \tilde{q}_d^A$ and the definition of $\tilde{\varphi}_d^A$ in (8).

Using the aggregate labor constraint we can rewrite the real consumption index as:

$$Q = L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} + f_d + \frac{f_e}{1 - G(\varphi_d^A)}\right]^{-\sigma/(\sigma-1)} \tilde{q}_d^A. \tag{111}$$

The social planner chooses the cutoff φ_d^A and quantity $\tilde{q}_d^A$ to maximize this consumption index. The trade-off faced by the social planner is that a lower productivity cutoff reduces expected entry costs conditional on successful entry, and thereby releases more labor for production. But this lower productivity cutoff involves firms of lower productivities producing, which reduces expected output conditional on successful entry. The first-order condition for $\tilde{q}_d^A$ yields:

$$\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} = (\sigma - 1)\left[f_d + \frac{f_e}{1 - G(\varphi_d^A)}\right]. \tag{112}$$

The first-order condition for φ_d^A is:

$$\frac{\frac{d\tilde{\varphi}_d^A}{d\varphi_d^A}\tilde{q}_d^A}{\left(\tilde{\varphi}_d^A\right)^2} = \frac{g\left(\varphi_d^A\right) f_e}{\left[1 - G\left(\varphi_d^A\right)\right]^2}.$$

Noting that:

$$\frac{d\tilde{\varphi}_d^A}{d\varphi_d^A} = \frac{1}{\sigma - 1}\left[\left(\tilde{\varphi}_d^A\right)^{\sigma-1} - \left(\varphi_d^A\right)^{\sigma-1}\right] \frac{g\left(\varphi_d^A\right)}{1 - G\left(\varphi_d^A\right)} \left(\tilde{\varphi}_d^A\right)^{1-(\sigma-1)},$$

the first-order condition for φ_d^A can be re-written as:

$$\frac{\tilde{q}_d^A \left(\tilde{\varphi}_d^A\right)^{-\sigma}}{\sigma - 1}\left[\left(\tilde{\varphi}_d^A\right)^{\sigma-1} - \left(\varphi_d^A\right)^{\sigma-1}\right] = \frac{f_e}{1 - G\left(\varphi^A\right)}.$$

Substituting the optimal quantity $\tilde{q}_d^A$ from (112) delivers the planner's solution for the optimal cutoff φ_d^A:

$$\left[1 - G(\varphi_d^A)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1} - 1\right] f_d = f_e, \tag{113}$$

which corresponds to the free entry condition in the market economy (6).

Therefore, the social planner chooses the same productivity cutoff φ_d^A (and the same values of all other endogenous variables) as in the market equilibrium of the heterogeneous firm model, which implies that the market equilibrium is efficient. In the heterogeneous firm model, changes in fixed costs and other parameters induce the social planner to change the productivity cutoff φ_d^A in addition to the output of a firm with weighted average productivity $\tilde{q}_d^A$ (and hence the mass of entrants M_e). In contrast, in the homogeneous firm model, productivity is constant by assumption, and hence changes in fixed costs and other parameters only induce changes in the mass of producing firms and output per firm.

8.2 Open Economy

The open economy real consumption index for the representative consumer is:

$$Q=\left[M_e\int_{\varphi_d^T}^{\varphi_{\max}}q_d\left(\varphi\right)^{(\sigma-1)/\sigma}\mathrm{d}G\left(\varphi\right)+M_e\int_{\varphi_x^T}^{\varphi_{\max}}\left(\frac{q_x\left(\varphi\right)}{\tau}\right)^{(\sigma-1)/\sigma}\mathrm{d}G\left(\varphi\right)\right]^{\sigma/(\sigma-1)}. \tag{114}$$

The social planner chooses the productivity cutoffs φ_d^T and φ_x^T, the output levels $q_d(\varphi)$ for the domestic market for all producing firms $\varphi\geq\varphi_d^T$, the output levels $q_x\left(\varphi\right)$ for the export market for all exporting firms $\varphi\geq\varphi_x^T$, and the mass of entrants M_e to maximize Q subject to the aggregate labor constraint:

$$L=M_e\left\{\begin{array}{l}\int_{\varphi_d^T}^{\varphi_{\max}}\frac{q_d(\varphi)}{\varphi}\mathrm{d}G\left(\varphi\right)+\int_{\varphi_x^T}^{\varphi_{\max}}\frac{q_x(\varphi)}{\varphi}\mathrm{d}G\left(\varphi\right)\\ +\left[1-G(\varphi_d^T)\right]f_d+\left[1-G(\varphi_x^T)\right]f_x+f_e\end{array}\right\},$$

where the social planner faces the same productivity distribution $G(\varphi)$ and entry cost f_e per firm as in the market allocation.

The planner chooses the output levels $q_d(\varphi)$ and $q_x\left(\varphi\right)$ to equate the marginal rates of transformations and marginal rates of substitution for firms with different productivities:

$$\frac{q_d(\varphi_1)}{q_d(\varphi_2)}=\frac{q_x(\varphi_1)}{q_x(\varphi_2)}=\left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

$$\frac{q_x(\varphi_1)}{q_d(\varphi_2)}=\tau^{1-\sigma}\left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma},$$

which are the same relationships between relative quantities and relative productivities as in the market equilibrium.

Using these relationships, we can rewrite the consumption index Q and the aggregate labor constraint as a function of the mass of firms serving each market M_t and the output level $\tilde{q}_t^T\equiv q_d(\tilde{\varphi}_t^T)$ of a firm with a weighted average productivity $\tilde{\varphi}_t^T$:

$$Q=M_t^{\sigma/(\sigma-1)}\tilde{q}_t^T,$$

$$\begin{aligned}L&=M_t\left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T}+\frac{\left[1-G(\varphi_d^T)\right]f_d+\left[1-G(\varphi_x^T)\right]f_x+f_e}{\left[1-G(\varphi_d^T)\right]+\left[1-G(\varphi_x^T)\right]}\right],\\&=M_t\left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T}+\frac{1}{1+\chi}\left(f_d+\chi f_x+\frac{f_e}{1-G(\varphi_d^T)}\right)\right].\end{aligned}$$

The mass of firms serving each market (M_t) and weighted average productivity ($\tilde{\varphi}_t^T$) are defined as follows:

$$M_t=\left[1+\chi\right]M=\left[1+\chi\right]\left[1-G\left(\varphi_d^T\right)\right]M_e, \tag{115}$$

$$\begin{aligned}\tilde{\varphi}_t^T&=\left\{\frac{\left[1-G\left(\varphi_d^T\right)\right]\left(\tilde{\varphi}_d^T\right)^{\sigma-1}+\left[1-G\left(\varphi_x^T\right)\right]\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]}\right\}^{1/(\sigma-1)},\\&=\left\{\frac{1}{1+\chi}\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1}+\chi\tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right]\right\}^{1/(\sigma-1)}.\end{aligned} \tag{116}$$

where χ is the proportion of exporting firms:

$$\chi=\frac{1-G\left(\varphi_x^T\right)}{1-G\left(\varphi_d^T\right)}.$$

Using the aggregate labor constraint, we can rewrite the real consumption index as:

$$Q = L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} + \frac{\left[1 - G(\varphi_d^T)\right] f_d + \left[1 - G(\varphi_x^T)\right] f_x + f_e}{\left[1 - G(\varphi_d^T)\right] + \left[1 - G(\varphi_x^T)\right]} \right]^{-\sigma/(\sigma-1)} \tilde{q}_t^T$$

$$Q = L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} + \frac{1}{1+\chi} \left(f_d + \chi f_x + \frac{f_e}{1 - G(\varphi_d^T)} \right) \right]^{-\sigma/(\sigma-1)} \tilde{q}_t^T. \tag{117}$$

The social planner chooses the productivity cutoffs φ_d^T and φ_x^T and quantity $\tilde{q}_t^T$ to maximize this consumption index. The trade-offs faced by the social planner are as follows. A lower domestic cutoff again reduces expected entry costs conditional on successful entry, and thereby releases more labor for production. But this lower domestic cutoff involves firms of lower productivities producing, which reduces expected output conditional on successful entry. A lower export cutoff for a given domestic cutoff increases the probability of exporting, which uses more labor in the fixed costs of exporting and reduces expected output conditional on exporting. But this lower export cutoff also increases the fraction of foreign varieties available to domestic consumers. The first-order condition for $\tilde{q}_t^T$ yields:

$$\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = (\sigma - 1) \left[\frac{\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e}{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]} \right],$$

$$\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = \frac{\sigma - 1}{1+\chi} \left[f_d + \chi f_x + \frac{f_e}{\left[1 - G\left(\varphi_d^T\right)\right]} \right],$$

$$\left[1 - G(\varphi_d^T)\right] \left[\frac{(1+\chi)\, \tilde{q}_t^T}{(\sigma-1)\, \tilde{\varphi}_t^T} - f_d - \chi f_x \right] = f_e, \tag{118}$$

which equates the expected profits from entry to the sunk entry cost, because $\frac{(1+\chi)\tilde{q}_t^T}{(\sigma-1)\tilde{\varphi}_t^T} = \frac{(1+\chi)\tilde{r}_t}{\sigma}$ is expected variable profits conditional on successful entry.

The first-order condition for φ_d^T requires:

$$-\frac{\tilde{q}_t^T \frac{\mathrm{d}\tilde{\varphi}_t^T}{\mathrm{d}\varphi_d^T}}{\left(\tilde{\varphi}_t^T\right)^2} - \frac{\left[1 - G\left(\varphi_x^T\right)\right] g\left(\varphi_d^T\right) f_d}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} + \frac{\left[1 - G\left(\varphi_x^T\right)\right] g\left(\varphi_d^T\right) f_x}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} + \frac{g\left(\varphi_d^T\right) f_e}{\left\{\left[1 - G\left(\varphi_d^T\right)\right] + \left[1 - G\left(\varphi_x^T\right)\right]\right\}^2} = 0.$$

Using the first-order condition for $\tilde{q}_t^T$ and noting that:

$$\frac{\mathrm{d}\tilde{\varphi}_t^T}{\mathrm{d}\varphi_d^T} = \frac{1}{\sigma-1} \left[\begin{array}{c} -\frac{\left[1-G\left(\varphi_x^T\right)\right] g\left(\varphi_d^T\right)\left(\tilde{\varphi}_d^T\right)^{\sigma-1}}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2} + \frac{\left[\left(\tilde{\varphi}_d^T\right)^{\sigma-1}-\left(\varphi_d^T\right)^{\sigma-1}\right] g\left(\varphi_d^T\right)}{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]} \\ +\frac{\left[1-G\left(\varphi_x^T\right)\right] g\left(\varphi_d^T\right) \tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2} \end{array} \right] \left(\varphi_t^T\right)^{1-(\sigma-1)},$$

the first-order condition for φ_d^T can we written as follows:

$$\left(\frac{\varphi_d^T}{\tilde{\varphi}_t^T}\right)^{\sigma-1} \frac{1}{\sigma-1} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = \frac{r_d\left(\varphi_d^T\right)}{\sigma} = f_d, \tag{119}$$

which corresponds to the domestic cutoff condition for the open economy market equilibrium (15).

The first-order condition for φ_x^T requires:

$$-\frac{\tilde{q}_t^T \frac{\mathrm{d}\tilde{\varphi}_t^T}{\mathrm{d}\varphi_x^T}}{\left(\tilde{\varphi}_t^T\right)^2}+\frac{\left[1-G\left(\varphi_d^T\right)\right] g\left(\varphi_x^T\right) f_d}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2}-\frac{\left[1-G\left(\varphi_d^T\right)\right] g\left(\varphi_x^T\right) f_x}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2}$$
$$+\frac{g\left(\varphi_x^T\right) f_e}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2}=0.$$

Using the first-order condition for $\tilde{q}_t^T$ and noting that:

$$\frac{\mathrm{d}\tilde{\varphi}_t^T}{\mathrm{d}\varphi_x^T}=\frac{1}{\sigma-1}\left[\begin{array}{c}\frac{\left[1-G\left(\varphi_d^T\right)\right] g\left(\varphi_x^T\right)\left(\tilde{\varphi}_d^T\right)^{\sigma-1}}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2}-\frac{\left[1-G\left(\varphi_d^T\right)\right] g\left(\varphi_x^T\right) \tau^{1-\sigma}\left(\tilde{\varphi}_x^T\right)^{\sigma-1}}{\left\{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]\right\}^2} \\ +\frac{\tau^{1-\sigma}\left[\left(\tilde{\varphi}_x^T\right)^{\sigma-1}-\left(\varphi_x^T\right)^{\sigma-1}\right] g\left(\varphi_x^T\right)}{\left[1-G\left(\varphi_d^T\right)\right]+\left[1-G\left(\varphi_x^T\right)\right]}\end{array}\right]\left(\varphi_t^T\right)^{1-(\sigma-1)},$$

the first-order condition for φ_x^T can be expressed as:

$$\left(\frac{\varphi_x^T}{\tilde{\varphi}_t^T}\right)^{\sigma-1} \frac{\tau^{1-\sigma}}{\sigma-1} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T}=\frac{r_x\left(\varphi_x^T\right)}{\sigma}=f_x, \tag{120}$$

which corresponds to the export cutoff condition for the open economy market equilibrium (16).

From the two cutoff conditions (119) and (120), the relationship between the domestic and export cutoffs is the same as in the open economy market equilibrium:

$$\varphi_x^T=\tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^T,$$

and the free entry condition (118) can be written in the same form as in the open economy market equilibrium (19):

$$\left[1-G(\varphi_d^T)\right]\left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1}-1\right] f_d+\left[1-G(\varphi_x^T)\right]\left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1}-1\right] f_x=f_e. \tag{121}$$

Since the free entry, domestic cutoff and export cutoff conditions for the social planner are the same as those for the open economy market equilibrium, the social planner chooses the same productivity cutoffs φ_d^T and φ_x^T (and the same value of all other endogenous variables) as in the open economy market equilibrium of the heterogeneous firm model. Hence the open economy market equilibrium of the heterogeneous firm model is efficient.

8.3 Open Versus Closed Economy

We choose the degenerate productivity distribution in the homogeneous firm model so that the two models yield the same values of all aggregate variables (including welfare) in the closed economy. Furthermore, it is technologically feasible for the social planner to choose the same domestic productivity cutoff in the open economy as in the closed economy ($\varphi_d^T=\varphi_d^A$) and to choose all firms to export ($\varphi_x^T=\varphi_d^T$). In this hypothetical allocation, the heterogeneous and homogeneous firm models would also generate the same values of all aggregate variables (including welfare) in the open economy.

However, comparing the closed and open economy free entry conditions (113) and (121), the social planner in general chooses a different domestic productivity cutoff in the open economy than in the closed economy ($\varphi_d^T \neq \varphi_d^A$) and in general restricts exporting to a proper subset of more productive firms ($\varphi_d^T<\varphi_x^T<\infty$). Since the social planner chooses different productivity cutoffs in the open economy when it is technologically feasible to choose the same productivity cutoffs as in the closed economy, revealed preference implies that these different productivity cutoffs must yield at least as high level of welfare as the hypothetical allocation with the same productivity cutoffs. Furthermore, the social planner's objective (117) is globally concave in $\{\varphi_d^T, \varphi_x^T, \tilde{q}_t^T\}$, which implies that the different productivity cutoffs must yield strictly higher welfare than the hypothetical allocation with the same productivity cutoffs. Since the social planner's choice corresponds to the open economy equilibrium of the heterogeneous firm model, and the hypothetical allocation corresponds to the open economy equilibrium of the homogeneous firm model, it follows that open economy welfare is strictly higher in the heterogeneous firm model than in the homogeneous firm model whenever the productivity cutoffs differ in the open and closed economies.

This revealed preference argument implies larger welfare gains in the heterogeneous firm model, both when there is selection into export markets $(\tau (f_x/f_d)^{1/(\sigma-1)} > 1$ and $\varphi_x^T > \varphi_d^T)$, and when all firms export $(\tau (f_x/f_d)^{1/(\sigma-1)} \leq 1$ and $\varphi_x^T = \varphi_d^T)$, as long as fixed exporting costs are positive $(f_x > 0)$. Comparing the free entry conditions in the open and closed economies (113) and (121), the social planner chooses different domestic productivity cutoffs in the open and closed economies $(\varphi_d^T \neq \varphi_d^A)$ for positive fixed exporting costs. The reason is that the social planner's objective in the open economy (117) features a different combination of fixed and variable costs from her objective in the closed economy (111). The social planner's optimal response to these different fixed and variable costs is to change the range of productivities for which firms serve each market as well as the mass of firms and average output per firm. Thus the greater welfare gains in the heterogeneous firm model reflect the presence of an additional adjustment margin (the firm productivity ranges) relative to the homogeneous firm model. In the heterogeneous firm model, the social planner can allocate low-productivity firms to serve only the domestic market and reallocate resources to higher-productivity exporting firms. Therefore the welfare that the social planner can achieve in a model with this additional adjustment margin must be at least as high (and in general higher) than in a model without it.

9 Alternative Revealed Preference Derivation

In this section, we provide a complementary alternative derivation of the result that the market equilibrium of the heterogeneous firm model is efficient, which is used as part of our revealed preference argument above. This alternative derivation formulates the social planner's problem as a Lagrangian and uses the first-order conditions for this Lagrangian to show that the socially optimal allocation replicates the allocation in the market equilibrium.

9.1 Closed Economy

9.1.1 Social Planner's Problem

We first show that in the closed economy the social planner chooses the same values of $\{h(\varphi), \varphi_d^A, M_e, Q\}$ as in the market equilibrium, where $h(\varphi)$ denotes variable labor input. The social planner's problem is to maximize the welfare of the representative consumer subject to the constraints of the entry and production technologies. The welfare of the representative consumer is determined by the real consumption index:

$$Q = \left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} (\varphi h(\varphi))^{\rho} \, \mathrm{d}G(\varphi) \right]^{\frac{1}{\rho}}. \tag{122}$$

where $\sigma = \frac{1}{1-\rho}$. Therefore the social planner solves the following constrained maximization problem:

$$\max_{\left\{h(\cdot),\varphi_d^A,M_e\right\}} \left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} \left(\varphi h\left(\varphi\right)\right)^{\rho} \mathrm{d}G\left(\varphi\right)\right]^{\frac{1}{\rho}} - \mu \left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d\right)\right] f_d + M_e f_e - L\right],$$

where μ is the multiplier on the social planner's constraint.

9.1.2 First-order Conditions

The first-order condition for M_e is :

$$\frac{1}{\rho M_e} Q - \frac{\mu}{M_e}\left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e\right] = 0. \tag{123}$$

The first-order condition for $h\left(\varphi\right)$ is:

$$M_e \varphi^{\rho} h\left(\varphi\right)^{\rho-1} g\left(\varphi\right) Q^{1-\rho} - \mu M_e g\left(\varphi\right) = 0. \tag{124}$$

The first-order condition for φ_d^A is:

$$-\frac{M_e}{\rho}\left(\varphi_d^A h\left(\varphi_d\right)\right)^{\rho} g\left(\varphi_d\right) Q^{1-\rho} + \mu M_e h\left(\varphi_d^A\right) g\left(\varphi_d^A\right) + \mu g\left(\varphi_d^A\right) M_e f_d = 0. \tag{125}$$

The first-order condition for μ is:

$$\left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e - L\right] = 0. \tag{126}$$

9.1.3 Lagrange Multiplier

Using the first-order condition for μ (126), the first-order condition for M_e (123) becomes:

$$\frac{1}{\rho M_e} Q - \frac{\mu}{M_e} L = 0,$$

$$\mu = \frac{Q}{\rho L}. \tag{127}$$

9.1.4 Employment

Using the first-order condition for μ (126), the first-order condition for $h\left(\varphi\right)$ (124) can be written:

$$h\left(\varphi\right) = \varphi^{\frac{\rho}{1-\rho}} Q^{-\frac{\rho}{1-\rho}} \left(\rho L\right)^{\frac{1}{1-\rho}}. \tag{128}$$

or equivalently:

$$h\left(\varphi\right) = \varphi^{\sigma-1} Q^{-(\sigma-1)} \left(\frac{\sigma-1}{\sigma} L\right)^{\sigma}, \tag{129}$$

9.1.5 Labor Market Clearing

Using the solution for $h(\varphi)$ from (128) in the first-order condition for μ (126), the labor market clearing condition can be written as follows:

$$L = M_e \int_{\varphi_d^A}^{\varphi_{\max}} h(\varphi)\, \mathrm{d}G(\varphi) + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e.$$

$$L = M_e Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[1 - G\left(\varphi_d^A\right)\right] \left(\tilde{\varphi}_d^A\right)^{\sigma-1} + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e. \tag{130}$$

$$\left(\tilde{\varphi}_d^A\right)^{\sigma-1} = \int_{\varphi_d^A}^{\varphi_{\max}} \varphi^{\sigma-1} \frac{\mathrm{d}G(\varphi)}{1 - G\left(\varphi_d^A\right)}.$$

9.1.6 Real Consumption Index and Mass of Firms

Using the solution for $h(\varphi)$ from (128) in the real consumption index (122) we get:

$$Q = M_e^{\frac{1}{\sigma-1}} \rho L \left[1 - G\left(\varphi_d^A\right)\right]^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A. \tag{131}$$

Substituting this expression for Q into the labor market clearing condition (130), we obtain the following expression for the mass of entrants as a function of φ_d^A and parameters:

$$M_e = \frac{(1-\rho) L}{\left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e}. \tag{132}$$

Using this result in (131), the real consumption index can also be written in terms of φ_d^A and parameters:

$$Q = \frac{\rho (1-\rho)^{\frac{1}{\sigma-1}} L^{\frac{\sigma}{\sigma-1}} \left[1 - G\left(\varphi_d^A\right)\right]^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A}{\left[\left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e\right]^{\frac{1}{\sigma-1}}}. \tag{133}$$

9.1.7 Productivity Cutoff Condition

Using the first-order condition for μ (126) and the solution for $h(\varphi)$ in (128), the first-order condition for φ_d^A can be written as:

$$\frac{1}{\sigma-1} \left(\varphi_d^A\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} = f_d. \tag{134}$$

9.1.8 Free Entry

To derive the analogue of the free entry condition for the social planner, note that the labor market clearing condition (130) gives us one expression for L/M_e:

$$\frac{L}{M_e} = Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[1 - G\left(\varphi_d^A\right)\right] \left(\tilde{\varphi}_d^A\right)^{\sigma-1} + \left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e,$$

while the mass of firms (132) gives us another expression for L/M_e:

$$\frac{L}{M_e} = \frac{\left[1 - G\left(\varphi_d^A\right)\right] f_d + f_e}{1-\rho} = \sigma \left[1 - G\left(\varphi_d^A\right)\right] f_d + \sigma f_e.$$

Equating these two expressions, we obtain the analogue of the free entry condition for the social planner:

$$\frac{1}{\sigma-1} \left[1 - G\left(\varphi_d^A\right)\right] \left(\tilde{\varphi}_d^A\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} - \left[1 - G\left(\varphi_d^A\right)\right] f_d = f_e. \tag{135}$$

Using the productivity cutoff condition (134) to substitute for $\frac{1}{\sigma-1}Q^{-(\sigma-1)}(\rho L)^{\sigma}$, the analogue of the free entry condition becomes:

$$f_d\left[1-G\left(\varphi_d^A\right)\right]\left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A}\right)^{\sigma-1}-1\right]=f_e. \tag{136}$$

The social planner's choices of $\{h(\varphi),\, \varphi_d^A,\, M_e,\, Q\}$ are determined by (129), (136), (132) and (133). This is the same system of equations that characterizes the open economy market equilibrium. It follows that the welfare-maximizing social planner chooses the same allocation $\{h(\varphi),\, \varphi_d^A,\, M_e,\, Q\}$ as the market equilibrium and that the market equilibrium is efficient.

9.2 Open Economy

9.2.1 Social Planner's Problem

We now show that in the open economy the social planner chooses the same values of $\{h_d(\varphi),\, h_x(\varphi),\, \varphi_d^T,\, \varphi_x^T,\, M_e,\, Q\}$ as in the market equilibrium, where $h_d(\varphi)$ denotes variable labor input for the domestic market and $h_x(\varphi)$ denotes variable labor input for the export market. The social planner's problem is to maximize the joint welfare of the two countries subject to the constraints of the entry and production technologies. Since the two countries are symmetric, the social planner maximizes the welfare of the representative consumer in each country, which is determined by the real consumption index:

$$Q=\left[M_e\int_{\varphi_d^T}^{\varphi_{\max}}\left(\varphi h_d(\varphi)\right)^{\rho}\mathrm{d}G(\varphi)+M_e\tau^{-\rho}\int_{\varphi_x^T}^{\varphi_{\max}}\left(\varphi h_x(\varphi)\right)^{\rho}\mathrm{d}G(\varphi)\right]^{\frac{1}{\rho}}. \tag{137}$$

Therefore the social planner solves the following constrained maximization problem:

$$\begin{aligned}\max_{\left\{h_d(\cdot),h_x(\cdot),\varphi_d^T,\varphi_x^T,M_e\right\}}&\left[M_e\int_{\varphi_d^T}^{\varphi_{\max}}\left(\varphi h_d(\varphi)\right)^{\rho}\mathrm{d}G(\varphi)+M_e\tau^{-\rho}\int_{\varphi_x^T}^{\varphi_{\max}}\left(\varphi h_x(\varphi)\right)^{\rho}\mathrm{d}G(\varphi)\right]^{\frac{1}{\rho}}\\ &-\mu\left[\begin{array}{c}M_e\int_{\varphi_d^T}^{\varphi_{\max}}h_d(\varphi)\,\mathrm{d}G(\varphi)+M_e\int_{\varphi_x^T}^{\varphi_{\max}}h_x(\varphi)\,\mathrm{d}G(\varphi)+M_e\left[1-G\left(\varphi_d^T\right)\right]f_d\\ +M_e\left[1-G\left(\varphi_x^T\right)\right]f_x+M_ef_e-L\end{array}\right],\end{aligned}$$

where μ is the multiplier on the constraint.

9.2.2 First-order Conditions

The first-order condition for M_e is :

$$\frac{1}{\rho M_e}Q-\frac{\mu}{M_e}\left[\begin{array}{c}M_e\int_{\varphi_d^T}^{\varphi_{\max}}h_d(\varphi)\,\mathrm{d}G(\varphi)+M_e\int_{\varphi_x^T}^{\varphi_{\max}}h_x(\varphi)\,\mathrm{d}G(\varphi)+M_e\left[1-G\left(\varphi_d^T\right)\right]f_d\\ +M_e\left[1-G\left(\varphi_x^T\right)\right]f_x+M_ef_e\end{array}\right]=0. \tag{138}$$

The first-order condition for $h_d(\varphi)$ is:

$$M_e\varphi^{\rho}h_d(\varphi)^{\rho-1}g(\varphi)\,Q^{1-\rho}-\mu M_eg(\varphi)=0. \tag{139}$$

The first-order condition for $h_x(\varphi)$ is:

$$\tau^{-\rho}M_e\varphi^{\rho}h_x(\varphi)^{\rho-1}g(\varphi)\,Q^{1-\rho}-\mu M_eg(\varphi)=0. \tag{140}$$

The first-order condition for φ_d^T is:

$$-\frac{M_e}{\rho}\left(\varphi_d^Th_d\left(\varphi_d^T\right)\right)^{\rho}g\left(\varphi_d^T\right)Q^{1-\rho}+\mu M_eh_d\left(\varphi_d^T\right)g\left(\varphi_d^T\right)+\mu g\left(\varphi_d^T\right)M_ef_d=0. \tag{141}$$

The first-order condition for φ_x^T is:

$$-\frac{M_e}{\rho}\tau^{-\rho}\left(\varphi_x^T h_d\left(\varphi_x^T\right)\right)^{\rho} g\left(\varphi_x^T\right) Q^{1-\rho} + \mu M_e h_x\left(\varphi_x^T\right) g\left(\varphi_x^T\right) + \mu g\left(\varphi_x^T\right) M_e f_x = 0. \tag{142}$$

The first-order condition for μ is:

$$\left[\begin{array}{c} M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d \\ +M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e - L \end{array}\right] = 0. \tag{143}$$

9.2.3 Lagrange Multiplier

Using the first-order condition for μ (143), the first-order condition for M_E (138) becomes:

$$\frac{1}{\rho M_e}Q - \frac{\mu}{M_e}L = 0,$$

$$\mu = \frac{Q}{\rho L}. \tag{144}$$

9.2.4 Employment

Using the first-order condition for μ (143), the first-order condition for $h_d\left(\varphi\right)$ (139) can be written:

$$h_d\left(\varphi\right) = \varphi^{\frac{\rho}{1-\rho}} Q^{-\frac{\rho}{1-\rho}} \left(\rho L\right)^{\frac{1}{1-\rho}}. \tag{145}$$

or equivalently:

$$h_d\left(\varphi\right) = \varphi^{\sigma-1} Q^{-(\sigma-1)} \left(\frac{\sigma-1}{\sigma}L\right)^{\sigma}, \tag{146}$$

Using the first-order condition for μ (143), the first-order condition for $h_x\left(\varphi\right)$ (140) can be written:

$$h_x\left(\varphi\right) = \tau^{-\frac{\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} Q^{-\frac{\rho}{1-\rho}} \left(\rho L\right)^{\frac{1}{1-\rho}}. \tag{147}$$

or equivalently:

$$h_x\left(\varphi\right) = \tau^{-(\sigma-1)} \varphi^{\sigma-1} Q^{-(\sigma-1)} \left(\frac{\sigma-1}{\sigma}L\right)^{\sigma}, \tag{148}$$

9.2.5 Labor Market Clearing

Using the solutions for $h_d\left(\varphi\right)$ and $h_x\left(\varphi\right)$ from (145) and (147) in the first-order condition for μ (143), the labor market clearing condition can be written as follows:

$$\begin{aligned} L = {} & M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d \\ & + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e. \end{aligned}$$

$$\begin{aligned} L = {} & M_e Q^{-(\sigma-1)} \left(\rho L\right)^{\sigma} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right] \\ & + M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e, \end{aligned} \tag{149}$$

where

$$\left(\tilde{\varphi}_d^T\right)^{\sigma-1} = \int_{\varphi_d^T}^{\varphi_{\max}} \varphi^{\sigma-1} \frac{\mathrm{d}G\left(\varphi\right)}{1 - G\left(\varphi_d^T\right)}, \qquad \left(\tilde{\varphi}_x^T\right)^{\sigma-1} = \int_{\varphi_x^T}^{\varphi_{\max}} \varphi^{\sigma-1} \frac{\mathrm{d}G\left(\varphi\right)}{1 - G\left(\varphi_x^T\right)}.$$

9.2.6 Real Consumption Index and Mass of Firms

Using the solutions for $h_d(\varphi)$ and $h_x(\varphi)$ from (145) and (147) in the real consumption index (137) we get:

$$Q = M_e^{\frac{1}{\sigma-1}} \rho L \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}. \tag{150}$$

Substituting this expression for Q into the labor market clearing condition (149), we obtain the following expression for the mass of entrants as a function of φ_d^T, φ_x^T and parameters:

$$M_e = \frac{(1-\rho) L}{\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e}. \tag{151}$$

Using this result in (150), the real consumption index can also be written in terms of φ_d^T and parameters:

$$Q = \frac{\rho (1-\rho)^{\frac{1}{\sigma-1}} L^{\frac{\sigma}{\sigma-1}} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}}{\left[\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e \right]^{\frac{1}{\sigma-1}}}. \tag{152}$$

9.2.7 Productivity Cutoff Conditions

Using the first-order condition for μ (143) and the solution for $h_d(\varphi)$ in (145), the first-order condition for φ_d^T can be written as:

$$\frac{1}{\sigma-1} \left(\varphi_d^T\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} = f_d. \tag{153}$$

Using the first-order condition for μ (143) and the solution for $h_x(\varphi)$ in (147), the first-order condition for φ_x^T can be written as:

$$\frac{1}{\sigma-1} \tau^{-(\sigma-1)} \left(\varphi_x^T\right)^{\sigma-1} Q^{-(\sigma-1)} (\rho L)^{\sigma} = f_x. \tag{154}$$

Together these cutoff conditions imply that the relative productivity cutoffs satisfy:

$$\left(\frac{\varphi_x^T}{\varphi_d^T}\right)^{\sigma-1} = \tau^{\sigma-1} \frac{f_x}{f_d}. \tag{155}$$

9.2.8 Free Entry

To derive the analogue of the free entry condition for the social planner, note that the labor market clearing condition (149) gives us one expression for L/M_e:

$$\begin{aligned} \frac{L}{M_e} = {} & Q^{-(\sigma-1)} (\rho L)^{\sigma} \left[\left[1 - G\left(\varphi_d^T\right)\right] \left(\tilde{\varphi}_d^T\right)^{\sigma-1} + \tau^{-(\sigma-1)} \left[1 - G\left(\varphi_x^T\right)\right] \left(\tilde{\varphi}_x^T\right)^{\sigma-1} \right] \\ & + \left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e, \end{aligned}$$

while the mass of firms (151) gives us another expression for L/M_e:

$$\begin{aligned} \frac{L}{M_e} &= \frac{\left[1 - G\left(\varphi_d^T\right)\right] f_d + \left[1 - G\left(\varphi_x^T\right)\right] f_x + f_e}{1-\rho} \\ &= \sigma \left[1 - G\left(\varphi_d^T\right)\right] f_d + \sigma \left[1 - G\left(\varphi_x^T\right)\right] f_x + \sigma f_e. \end{aligned}$$

Equating these two expressions, we obtain the analogue of the free entry condition for the social planner:

$$\frac{1}{\sigma-1}Q^{-(\sigma-1)}\left(\rho L\right)^{\sigma}\left[\left[1-G\left(\varphi_d^T\right)\right]\left(\tilde{\varphi}_d^T\right)^{\sigma-1}+\tau^{-(\sigma-1)}\left[1-G\left(\varphi_x^T\right)\right]\left(\tilde{\varphi}_x^T\right)^{\sigma-1}\right] \\ -\left[1-G\left(\varphi_d^T\right)\right]f_d-\left[1-G\left(\varphi_x^T\right)\right]f_x=f_e.$$

Using the productivity cutoff conditions (153) and (154) to substitute for $\frac{1}{\sigma-1}Q^{-(\sigma-1)}\left(\rho L\right)^{\sigma}$, the analogue of the free entry condition can be written as:

$$f_d\left[1-G\left(\varphi_d^T\right)\right]\left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T}\right)^{\sigma-1}-1\right]+f_x\left[1-G\left(\varphi_x^T\right)\right]\left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T}\right)^{\sigma-1}-1\right]=f_e. \tag{156}$$

The social planner's choices of $\{h_d\left(\varphi\right), h_x\left(\varphi\right), \varphi_d^T, \varphi_x^T, M_e, Q\}$ are determined by (146), (148), (156), (155), (151) and (152). This is the same system of equations that characterizes the open economy market equilibrium. It follows that the welfare-maximizing social planner chooses the same allocation $\{h_d\left(\varphi\right), h_x\left(\varphi\right), \varphi_d^T, \varphi_x^T, M_e, Q\}$ as the market equilibrium and that the market equilibrium is efficient.

10 Variable Elasticity of Substitution Preferences

With Variable Elasticity of Substitution (VES) preferences, the allocations chosen by the social planner and the market equilibrium are not in general the same. Again we choose the degenerate productivity distribution in the homogeneous firm model so that the two models have the same welfare and other aggregate variables for an initial value of trade costs. Following a change in trade costs, the heterogeneous firm model has the potential to generate higher welfare than the homogenous firm model, as long as adjustment along the additional margin of endogenous firm selection is similar in the market equilibrium and social optimum. We now provide a formal comparison of the allocations chosen by the social planner and the market equilibrium.

10.1 Social Planner Closed Economy VES

10.1.1 Social Planner's Problem

The social planner's problem is to maximize the welfare of the representative consumer subject to the constraints of the entry and production technologies. The real consumption index is:

$$Q^{\mathrm{SP}}=M_e\int_{\varphi_d^A}^{\varphi_{\max}}u\left[\varphi h\left(\varphi\right)\right]\mathrm{d}G\left(\varphi\right) \tag{157}$$

Therefore the social planner solves the following constrained maximization problem:

$$\max_{\left\{h(\cdot),\varphi_d^A,M_e\right\}} M_e\int_{\varphi_d^A}^{\varphi_{\max}}u\left[\varphi h\left(\varphi\right)\right]\mathrm{d}G\left(\varphi\right) \\ -\mu\left[M_e\int_{\varphi_d^A}^{\varphi_{\max}}h\left(\varphi\right)\mathrm{d}G\left(\varphi\right)+M_e\left[1-G\left(\varphi_d^A\right)\right]f_d+M_ef_e-L\right],$$

where μ is the multiplier on the social planner's constraint.

10.1.2 First-order Conditions

The first-order condition for M_e is:

$$\frac{1}{M_e}Q^{\mathrm{SP}} - \frac{\mu}{M_e}\left[M_e\int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right)\mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e\right] = 0. \tag{158}$$

The first-order condition for $h\left(\varphi\right)$ is:

$$M_e\varphi u'\left[\varphi h(\varphi)\right] g\left(\varphi\right) - \mu M_e g\left(\varphi\right) = 0. \tag{159}$$

The first-order condition for φ_d^A is:

$$-M_e u\left[\varphi_d^A h(\varphi_d^A)\right] g\left(\varphi_d^A\right) + \mu M_e h\left(\varphi_d^A\right) g\left(\varphi_d^A\right) + \mu g\left(\varphi_d^A\right) M_e f_d = 0. \tag{160}$$

The first-order condition for μ is:

$$\left[M_e\int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right)\mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e - L\right] = 0. \tag{161}$$

10.1.3 Lagrange Multiplier

Using the first-order condition for μ (161), the first-order condition for M_e (158) becomes:

$$\frac{1}{M_e}Q^{\mathrm{SP}} - \frac{\mu}{M_e}L = 0,$$

$$\mu = \frac{Q^{\mathrm{SP}}}{L}. \tag{162}$$

10.1.4 Employment

Using the first-order condition for μ (161), the first-order condition for $h\left(\varphi\right)$ (159) can be written:

$$\varphi u'\left[\varphi h(\varphi)\right] = \frac{Q^{\mathrm{SP}}}{L},$$

which determines relative employments as a function of relative productivities for any pair of varieties a and b:

$$\varphi_a u'\left[\varphi_a h(\varphi_a)\right] = \varphi_b u'\left[\varphi_b h(\varphi_b)\right].$$

We can therefore write the employment of each firm as a function of its productivity, the productivity of the least productive firm (φ_d^A) and the employment of the least productive firm (h_d^A):

$$h(\varphi) = h(\varphi, \varphi_d^A, h_d^A). \tag{163}$$

10.1.5 Labor Market Clearing

Using the first-order condition for μ (161), the labor market clearing condition can be written as follows:

$$L = M_e\int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right)\mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e,$$

which using employment (163) determines the mass of entrants as a function of the productivity (φ_d^A) and employment (h_d^A) of the least productive firm:

$$L = M_e\int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi, \varphi_d^A, h_d^A\right)\mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e,$$

$$M_e = M_e(\varphi_d^A, h_d^A), \tag{164}$$

which in turn determines the real consumption index as a function of the productivity (φ_d^A) and employment (h_d^A) of the least productive firm:

$$Q^{\text{SP}}(\varphi_d^A, h_d^A) = M_e(\varphi_d^A, h_d^A) \int_{\varphi_d^A}^{\varphi_{\max}} u\left[\varphi h(\varphi, \varphi_d^A, h_d^A)\right] \mathrm{d}G(\varphi). \tag{165}$$

10.1.6 Zero-profit Cutoff Productivity

Using the first-order condition for μ (161), the first-order condition for φ_d^A (160) can be written as:

$$u\left[\varphi_d^A h_d^A\right] - \frac{Q^{\text{SP}}(\varphi_d^A, h_d^A)}{L} h_d^A = \frac{Q^{\text{SP}}(\varphi_d^A, h_d^A)}{L} f_d,$$

which determines the employment of the least productive firm as a function of the cutoff productivity (φ_d^A):

$$h_d^A = h_d^A(\varphi_d^A). \tag{166}$$

To determine the cutoff productivity (φ_d^A), we return to the labor market clearing condition:

$$L = M_e(\varphi_d^A, h_d^A(\varphi_d^A)) \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi, \varphi_d^A, h_d^A(\varphi_d^A)\right) \mathrm{d}G(\varphi) + M_e(\varphi_d^A, h_d^A(\varphi_d^A)) \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e(\varphi_d^A, h_d^A(\varphi_d^A)) f_e,$$

where we have determined $h_d^A(\varphi_d^A)$ as a function of φ_d^A. Therefore the above labor market clearing condition determines the cutoff productivity φ_d^A.

10.2 Market Equilibrium Closed Economy VES

10.2.1 Market Allocation

The market allocation can be represented as the solution to the following constrained maximization problem. The objective corresponds to aggregate revenue:

$$Q^{\text{MK}} = M_e \int_{\varphi_d^A}^{\varphi_{\max}} u'\left[\varphi h(\varphi)\right] \varphi h(\varphi) \mathrm{d}G(\varphi) \tag{167}$$

The market allocation solves the following constrained maximization problem:

$$\max_{\{h(\cdot), \varphi_d^A, M_e\}} M_e \int_{\varphi_d^A}^{\varphi_{\max}} u'\left[\varphi h(\varphi)\right] \varphi h(\varphi) \mathrm{d}G(\varphi) - \mu \left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} h(\varphi) \mathrm{d}G(\varphi) + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e - L\right].$$

10.2.2 First-order Conditions

The first-order condition for M_e is:

$$\frac{1}{M_e} Q^{\text{MK}} - \frac{\mu}{M_e} \left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} h(\varphi) \mathrm{d}G(\varphi) + M_e \left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e\right] = 0. \tag{168}$$

The first-order condition for $h(\varphi)$ is:

$$M_e u'\left[\varphi h(\varphi)\right] \varphi g(\varphi) + M_e \varphi u''\left[\varphi h(\varphi)\right] \varphi h(\varphi) g(\varphi) - \mu M_e g(\varphi) = 0. \tag{169}$$

The first-order condition for φ_d^A is:

$$-M_e u'\left[\varphi_d^A h(\varphi_d^A)\right]\varphi_d^A h(\varphi_d^A) g\left(\varphi_d^A\right) + \mu M_e h\left(\varphi_d^A\right) g\left(\varphi_d^A\right) + \mu g\left(\varphi_d^A\right) M_e f_d = 0. \tag{170}$$

The first-order condition for μ is:

$$\left[M_e \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e - L\right] = 0. \tag{171}$$

10.2.3 Lagrange Multiplier

Using the first-order condition for μ (171), the first-order condition for M_e (168) becomes:

$$\frac{1}{M_e} Q^{\mathrm{MK}} - \frac{\mu}{M_e} L = 0,$$

$$\mu = \frac{Q^{\mathrm{MK}}}{L}. \tag{172}$$

10.2.4 Employment

Using the first-order condition for μ (171), the first-order condition for $h\left(\varphi\right)$ (169) can be written:

$$\varphi u'\left[\varphi h(\varphi)\right] + \varphi u''\left[\varphi h(\varphi)\right] \varphi h(\varphi) = \frac{Q^{\mathrm{MK}}}{L},$$

which determines relative employments as a function of relative productivities for any pair of varieties a and b:

$$\varphi_a u'\left[\varphi_a h(\varphi_a)\right] + \varphi_a u''\left[\varphi_a h(\varphi_a)\right] \varphi_a h(\varphi_a) = \varphi_b u'\left[\varphi_b h(\varphi_b)\right] + \varphi_b u''\left[\varphi_b h(\varphi_b)\right] \varphi_b h(\varphi_b).$$

We can therefore write the employment of each firm as a function of its productivity, the productivity of the least productive firm (φ_d^A) and the employment of the least productive firm (h_d^A):

$$h(\varphi) = h(\varphi, \varphi_d^A, h_d^A), \tag{173}$$

where the function $h(\cdot)$ differs between the market equilibrium here and the social optimum in the previous subsection.

10.2.5 Labor Market Clearing

Using the first-order condition for μ (171), the labor market clearing condition can be written as follows:

$$L = M_e \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e,$$

which using employment (173) determines the mass of entrants as a function of the productivity (φ_d^A) and employment (h_d^A) of the least productive firm:

$$L = M_e \int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi, \varphi_d^A, h_d^A\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^A\right)\right] f_d + M_e f_e,$$

$$M_e = M_e(\varphi_d^A, h_d^A), \tag{174}$$

where the function $M_e(\cdot)$ differs between the market equilibrium here and the social optimum in the previous subsection. Using this function for the mass of entrants, we can determine aggregate revenue as a function of the productivity (φ_d^A) and employment (h_d^A) of the least productive firm:

$$Q^{\mathrm{MK}}(\varphi_d^A, h_d^A) = M_e(\varphi_d^A, h_d^A) \int_{\varphi_d^A}^{\varphi_{\max}} u\left[\varphi h(\varphi, \varphi_d^A, h_d^A)\right] \mathrm{d}G\left(\varphi\right). \tag{175}$$

10.2.6 Zero-profit Cutoff Productivity

Using the first-order condition for μ (171), the first-order condition for φ_d^A (170) can be written as:

$$u'\left[\varphi_d^A h_d^A\right]\varphi_d^A h_d^A - \frac{Q^{\mathrm{MK}}(\varphi_d^A, h_d^A)}{L}h_d^A = \frac{Q^{\mathrm{MK}}(\varphi_d^A, h_d^A)}{L}f_d,$$

which determines the employment of the least productive firm as a function of the cutoff productivity (φ_d^A):

$$h_d^A = h_d^A(\varphi_d^A), \tag{176}$$

where h_d^A also differs between the market equilibrium here and the social optimum in the previous subsection. To determine the cutoff productivity (φ_d^A), we return to the labor market clearing condition:

$$L = M_e(\varphi_d^A, h_d^A(\varphi_d^A))\int_{\varphi_d^A}^{\varphi_{\max}} h\left(\varphi, \varphi_d^A, h_d^A(\varphi_d^A)\right)\mathrm{d}G\left(\varphi\right) + M_e(\varphi_d^A, h_d^A(\varphi_d^A))\left[1 - G\left(\varphi_d^A\right)\right]f_d + M_e(\varphi_d^A, h(\varphi_d^A))f_e,$$

where we have determined $h_d^A(\varphi_d^A)$ as a function of φ_d^A. Therefore the above labor market clearing condition determines the cutoff productivity φ_d^A, which again differs between the market equilibrium here and the social optimum in the previous subsection.

10.3 Social Planner Open Economy VES

10.3.1 Social Planner's Problem

The social planner's problem is to maximize the welfare of the representative consumer subject to the constraints of the entry and production technologies. The real consumption index is:

$$Q^{\mathrm{SP}} = M_e\int_{\varphi_d^T}^{\varphi_{\max}} u\left[\varphi h_d\left(\varphi\right)\right]\mathrm{d}G\left(\varphi\right) + M_e\int_{\varphi_x^T}^{\varphi_{\max}} u\left[\frac{\varphi h_x\left(\varphi\right)}{\tau}\right]\mathrm{d}G\left(\varphi\right). \tag{177}$$

Therefore the social planner solves the following constrained maximization problem:

$$\max_{\left\{h_d(\cdot), h_x(\cdot), \varphi_d^T, \varphi_x^T, M_e\right\}}\left[M_e\int_{\varphi_d^T}^{\varphi_{\max}} u\left[\varphi h_d\left(\varphi\right)\right]\mathrm{d}G\left(\varphi\right) + M_e\int_{\varphi_x^T}^{\varphi_{\max}} u\left[\frac{\varphi h_x\left(\varphi\right)}{\tau}\right]\mathrm{d}G\left(\varphi\right)\right]$$
$$-\mu\left[M_e\int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right)\mathrm{d}G\left(\varphi\right) + M_e\int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right)\mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^T\right)\right]f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right]f_x + M_e f_e - L\right],$$

where μ is the multiplier on the social planner's constraint.

10.3.2 First-order Conditions

The first-order condition for M_e is:

$$\frac{1}{M_e}Q^{\mathrm{SP}} - \frac{\mu}{M_e}\left[\begin{array}{c} M_e\int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right)\mathrm{d}G\left(\varphi\right) + M_e\int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right)\mathrm{d}G\left(\varphi\right) \\ +M_e\left[1 - G\left(\varphi_d^T\right)\right]f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right]f_x + M_e f_e \end{array}\right] = 0. \tag{178}$$

The first-order condition for $h_d\left(\varphi\right)$ is:

$$M_e\varphi u'\left[\varphi h_d(\varphi)\right]g\left(\varphi\right) - \mu M_e g\left(\varphi\right) = 0. \tag{179}$$

The first-order condition for $h_x(\varphi)$ is:

$$M_e \frac{\varphi}{\tau} u' \left[\frac{\varphi h_x(\varphi)}{\tau} \right] g(\varphi) - \mu M_e g(\varphi) = 0. \tag{180}$$

The first-order condition for φ_d^T is:

$$-M_e u \left[\varphi_d^T h_d(\varphi_d^T) \right] g\left(\varphi_d^T\right) + \mu M_e h_d\left(\varphi_d^T\right) g\left(\varphi_d^T\right) + \mu g\left(\varphi_d^T\right) M_e f_d = 0. \tag{181}$$

The first-order condition for φ_x^T is:

$$-M_e u \left[\frac{\varphi_x^T h_x(\varphi_x^T)}{\tau} \right] g\left(\varphi_x^T\right) + \mu M_e h_x\left(\varphi_x^T\right) g\left(\varphi_x^T\right) + \mu g\left(\varphi_x^T\right) M_e f_x = 0. \tag{182}$$

The first-order condition for μ is:

$$\left[\begin{array}{c} M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d(\varphi) \, \mathrm{d}G(\varphi) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x(\varphi) \, \mathrm{d}G(\varphi) \\ \\ +M_e \left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e \left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e - L \end{array} \right] = 0. \tag{183}$$

10.3.3 Lagrange Multiplier

Using the first-order condition for μ (183), the first-order condition for M_e (178) becomes:

$$\frac{1}{M_e} Q^{\mathrm{SP}} - \frac{\mu}{M_e} L = 0,$$

$$\mu = \frac{Q^{\mathrm{SP}}}{L}. \tag{184}$$

10.3.4 Employment

Using the first-order condition for μ (183), the first-order condition for $h_d(\varphi)$ (179) can be written:

$$\varphi u' \left[\varphi h_d(\varphi) \right] = \frac{Q^{\mathrm{SP}}}{L},$$

which determines relative domestic employments as a function of relative productivities for any pair of varieties a and b:

$$\varphi_a u' \left[\varphi_a h_d(\varphi_a) \right] = \varphi_b u' \left[\varphi_b h_d(\varphi_b) \right].$$

We can therefore write the domestic employment of each firm as a function of its productivity, the productivity of the least productive domestic firm (φ_d^T) and the domestic employment of the least productive firm (h_d^T):

$$h_d(\varphi) = h_d(\varphi, \varphi_d^T, h_d^T). \tag{185}$$

Using the first-order condition for μ (183), the first-order condition for $h_x(\varphi)$ (180) can be written:

$$\frac{\varphi}{\tau} u' \left[\frac{\varphi h_x(\varphi)}{\tau} \right] = \frac{Q^{\mathrm{SP}}}{L},$$

which determines relative export employments as a function of relative productivities for any pair of varieties a and b:

$$\frac{\varphi_a}{\tau} u' \left[\frac{\varphi_a h_x(\varphi_a)}{\tau} \right] = \frac{\varphi_b}{\tau} u' \left[\frac{\varphi_b h_x(\varphi_b)}{\tau} \right].$$

We can therefore write the export employment of each firm as a function of its productivity, the productivity of the least productive exporter (φ_x^T), the export employment of the least productive exporter (h_x^T), and variable trade costs:

$$h_x(\varphi) = h_x(\varphi, \varphi_x^T, h_x^T, \tau). \tag{186}$$

Note that the following relationship also holds:

$$\varphi_a u' \left[\varphi_a h_d(\varphi_a)\right] = \frac{\varphi_b}{\tau} u' \left[\frac{\varphi_b h_d(\varphi_b)}{\tau}\right].$$

10.3.5 Labor Market Clearing

Using the first-order condition for μ (183), the labor market clearing condition can be written as follows:

$$L = M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e,$$

which using domestic employment (185) and export employment (186) determines the mass of entrants as a function of the productivity (φ_d^T) and employment (h_d^T) of the least productive domestic firm, the productivity (φ_x^T) and employment (h_x^T) of the least productive exporter, and variable trade costs τ:

$$L = \left[\begin{array}{l} M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi, \varphi_d^T, h_d^T\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi, \varphi_x^T, h_x^T, \tau\right) \mathrm{d}G\left(\varphi\right) \\ \\ + M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e \end{array}\right],$$

$$M_e = M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau), \tag{187}$$

which in turn determines the real consumption index as a function of $\{\varphi_d^T,\ h_d^T,\ \varphi_x^T,\ h_x^T,\ \tau\}$:

$$Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) = \left[\begin{array}{l} M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_d^T}^{\varphi_{\max}} u\left[\varphi h_d(\varphi, \varphi_d^T, h_d^T)\right] \mathrm{d}G\left(\varphi\right) \\ \\ + M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_x^T}^{\varphi_{\max}} u\left[\frac{\varphi h_x(\varphi, \varphi_x^T, h_x^T, \tau)}{\tau}\right] \mathrm{d}G\left(\varphi\right) \end{array}\right]. \tag{188}$$

10.3.6 Zero-profit and Export Cutoff Productivities

Using the first-order condition for μ (183), the first-order condition for φ_d^T (181) can be written as:

$$u\left[\varphi_d^T h_d^T\right] - \frac{Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_d^T = \frac{Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} f_d.$$

Using the first-order condition for μ (183), the first-order condition for φ_x^T (182) can be written as:

$$u\left[\frac{\varphi_x^T h_x^T}{\tau}\right] - \frac{Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_x^T = \frac{Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} f_x,$$

Taking the ratio of these two productivity cutoff conditions, we obtain:

$$\frac{u\left[\varphi_d^T h_d^T\right] - \frac{Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_d^T}{u\left[\frac{\varphi_x^T h_x^T}{\tau}\right] - \frac{Q^{\mathrm{SP}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_x^T} = \frac{f_d}{f_x}.$$

The productivity levels and employment levels of the least productive domestic firm and exporter also must be such that the labor market clearing condition is satisfied:

$$L = \begin{bmatrix} M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi, \varphi_d^T, h_d^T\right) \mathrm{d}G\left(\varphi\right) + M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi, \varphi_x^T, h_x^T, \tau\right) \mathrm{d}G\left(\varphi\right) \\ \\ +M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)\left[1 - G\left(\varphi_x^T\right)\right] f_x \\ \\ +M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) f_e \end{bmatrix}.$$

10.4 Market Equilibrium Open Economy VES

10.4.1 Market Allocation

The market allocation can be represented as the solution to the following constrained maximization problem. The objective corresponds to aggregate revenue:

$$Q^{\mathrm{MK}} = M_e \int_{\varphi_d^T}^{\varphi_{\max}} u'\left[\varphi h_d\left(\varphi\right)\right] \varphi h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} u'\left[\frac{\varphi h_x\left(\varphi\right)}{\tau}\right] \frac{\varphi h_x\left(\varphi\right)}{\tau} \mathrm{d}G\left(\varphi\right). \tag{189}$$

Therefore the social planner solves the following constrained maximization problem:

$$\max_{\left\{h_d(\cdot), h_x(\cdot), \varphi_d^T, \varphi_x^T, M_e\right\}} \left[M_e \int_{\varphi_d^T}^{\varphi_{\max}} u'\left[\varphi h_d\left(\varphi\right)\right] \varphi h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} u'\left[\frac{\varphi h_x\left(\varphi\right)}{\tau}\right] \frac{\varphi h_x\left(\varphi\right)}{\tau} \mathrm{d}G\left(\varphi\right)\right]$$

$$-\mu\left[M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e - L\right].$$

10.4.2 First-order Conditions

The first-order condition for M_e is:

$$\frac{1}{M_e} Q^{\mathrm{MK}} - \frac{\mu}{M_e} \begin{bmatrix} M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right) \mathrm{d}G\left(\varphi\right) \\ \\ +M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e \end{bmatrix} = 0. \tag{190}$$

The first-order condition for $h_d\left(\varphi\right)$ is:

$$M_e u'\left[\varphi h_d(\varphi)\right] \varphi g\left(\varphi\right) + M_e \varphi u''\left[\varphi h_d(\varphi)\right] \varphi h_d\left(\varphi\right) g\left(\varphi\right) - \mu M_e g\left(\varphi\right) = 0. \tag{191}$$

The first-order condition for $h_x\left(\varphi\right)$ is:

$$M_e u'\left[\frac{\varphi h_x(\varphi)}{\tau}\right] \frac{\varphi}{\tau} g\left(\varphi\right) + M_e \frac{\varphi}{\tau} u''\left[\frac{\varphi h_x(\varphi)}{\tau}\right] \frac{\varphi h_x\left(\varphi\right)}{\tau} g\left(\varphi\right) - \mu M_e g\left(\varphi\right) = 0. \tag{192}$$

The first-order condition for φ_d^T is:

$$-M_e u'\left[\varphi_d^T h_d(\varphi_d^T)\right] \varphi_d^T h_d(\varphi_d^T) g\left(\varphi_d^T\right) + \mu M_e h_d\left(\varphi_d^T\right) g\left(\varphi_d^T\right) + \mu g\left(\varphi_d^T\right) M_e f_d = 0. \tag{193}$$

The first-order condition for φ_x^T is:

$$-M_e u'\left[\frac{\varphi_x^T h_x(\varphi_x^T)}{\tau}\right] \frac{\varphi_x^T h_x(\varphi_x^T)}{\tau} g\left(\varphi_x^T\right) + \mu M_e h_x\left(\varphi_x^T\right) g\left(\varphi_x^T\right) + \mu g\left(\varphi_x^T\right) M_e f_x = 0. \tag{194}$$

The first-order condition for μ is:

$$\begin{bmatrix} M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi\right) \mathrm{d}G\left(\varphi\right) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi\right) \mathrm{d}G\left(\varphi\right) \\ \\ +M_e\left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e\left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e - L \end{bmatrix} = 0. \tag{195}$$

10.4.3 Lagrange Multiplier

Using the first-order condition for μ (195), the first-order condition for M_e (190) becomes:

$$\frac{1}{M_e}Q^{\mathrm{MK}} - \frac{\mu}{M_e}L = 0,$$

$$\mu = \frac{Q^{\mathrm{MK}}}{L}. \tag{196}$$

10.4.4 Employment

Using the first-order condition for μ (195), the first-order condition for $h_d(\varphi)$ (191) can be written:

$$\varphi u'\left[\varphi h_d(\varphi)\right] + \varphi u''\left[\varphi h_d(\varphi)\right]\varphi h_d(\varphi) = \frac{Q^{\mathrm{MK}}}{L},$$

which determines relative domestic employments as a function of relative productivities for any pair of varieties a and b:

$$\varphi_a u'\left[\varphi_a h_d(\varphi_a)\right] + \varphi_a u''\left[\varphi_a h_d(\varphi_a)\right]\varphi_a h_d(\varphi_a) = \varphi_b u'\left[\varphi_b h_d(\varphi_b)\right] + \varphi_b u''\left[\varphi_b h_d(\varphi_b)\right]\varphi_b h_d(\varphi_b).$$

We can therefore write the domestic employment of each firm as a function of its productivity, the productivity of the least productive domestic firm (φ_d^T) and the domestic employment of the least productive firm (h_d^T):

$$h_d(\varphi) = h_d(\varphi, \varphi_d^T, h_d^T), \tag{197}$$

where the function $h_d(\cdot)$ differs between the market equilibrium here and the social optimum in the previous subsection. Using the first-order condition for μ (195), the first-order condition for $h_x(\varphi)$ (192) can be written:

$$\frac{\varphi}{\tau}u'\left[\frac{\varphi h_x(\varphi)}{\tau}\right] + \frac{\varphi}{\tau}u''\left[\frac{\varphi h_x(\varphi)}{\tau}\right]\frac{\varphi h_x(\varphi)}{\tau} = \frac{Q^{\mathrm{MK}}}{L},$$

which determines relative export employments as a function of relative productivities for any pair of varieties a and b:

$$\frac{\varphi_a}{\tau}u'\left[\frac{\varphi_a h_x(\varphi_a)}{\tau}\right] + \frac{\varphi_a}{\tau}u''\left[\frac{\varphi_a h_x(\varphi_a)}{\tau}\right]\frac{\varphi_a h_x(\varphi_a)}{\tau} = \frac{\varphi_b}{\tau}u'\left[\frac{\varphi_b h_x(\varphi_b)}{\tau}\right] + \frac{\varphi_b}{\tau}u''\left[\frac{\varphi_b h_x(\varphi_b)}{\tau}\right]\frac{\varphi_b h_x(\varphi_b)}{\tau}.$$

We can therefore write the export employment of each firm as a function of its productivity, the productivity of the least productive exporter (φ_x^T), the export employment of the least productive exporter (h_x^T), and variable trade costs:

$$h_x(\varphi) = h_x(\varphi, \varphi_x^T, h_x^T, \tau), \tag{198}$$

where the function $h_x(\cdot)$ differs between the market equilibrium here and the social optimum in the previous subsection. Note that the following relationship also holds:

$$\varphi_a u'\left[\varphi_a h_d(\varphi_a)\right] + \varphi_a u''\left[\varphi_a h_d(\varphi_a)\right]\varphi_a h_d(\varphi_a) = \frac{\varphi_b}{\tau}u'\left[\frac{\varphi_b h_d(\varphi_b)}{\tau}\right] + \frac{\varphi_b}{\tau}u''\left[\frac{\varphi_b h_d(\varphi_b)}{\tau}\right]\frac{\varphi_b h_d(\varphi_b)}{\tau}.$$

10.4.5 Labor Market Clearing

Using the first-order condition for μ (195), the labor market clearing condition can be written as follows:

$$L = M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d(\varphi)\, \mathrm{d}G(\varphi) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x(\varphi)\, \mathrm{d}G(\varphi) + M_e \left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e \left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e,$$

which using domestic employment (197) and export employment (198) determines the mass of entrants as a function of the productivity (φ_d^T) and employment (h_d^T) of the least productive domestic firm, and the productivity (φ_x^T) and employment (h_x^T) of the least productive exporter, and variable trade costs (τ):

$$L = \left[\begin{array}{l} M_e \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi, \varphi_d^T, h_d^T\right) \mathrm{d}G(\varphi) + M_e \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi, \varphi_x^T, h_x^T, \tau\right) \mathrm{d}G(\varphi) \\ \\ +M_e \left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e \left[1 - G\left(\varphi_x^T\right)\right] f_x + M_e f_e \end{array} \right],$$

$$M_e = M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau), \tag{199}$$

where the function $M_e(\cdot)$ also differs between the market equilibrium here and the social optimum in the previous subsection. Using this function for the mass of entrants, we can determine aggregate revenue as a function of $\{\varphi_d^T,\, h_d^T,\, \varphi_x^T,\, h_x^T,\, \tau\}$:

$$Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) = \left[\begin{array}{l} M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_d^T}^{\varphi_{\max}} u'\left[\varphi h_d(\varphi, \varphi_d^T, h_d^T)\right] \varphi h_d(\varphi, \varphi_d^T, h_d^T) \mathrm{d}G(\varphi) \\ \\ +M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_x^T}^{\varphi_{\max}} u'\left[\frac{\varphi h_x(\varphi, \varphi_x^T, h_x^T, \tau)}{\tau}\right] \frac{\varphi h_x(\varphi, \varphi_x^T, h_x^T, \tau)}{\tau} \mathrm{d}G(\varphi) \end{array} \right]. \tag{200}$$

10.4.6 Zero-profit and Export Cutoff Productivities

Using the first-order condition for μ (195), the first-order condition for φ_d^T (193) can be written as:

$$u'\left[\varphi_d^T h_d^T\right] \varphi_d^T h_d^T - \frac{Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_d^T = \frac{Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} f_d.$$

Using the first-order condition for μ (195), the first-order condition for φ_x^T (194) can be written as:

$$u'\left[\frac{\varphi_x^T h_x^T}{\tau}\right] \frac{\varphi_x^T h_x^T}{\tau} - \frac{Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_x^T = \frac{Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} f_x,$$

Taking the ratio of these two productivity cutoff conditions, we obtain:

$$\frac{u'\left[\varphi_d^T h_d^T\right] \varphi_d^T h_d^T - \frac{Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_d^T}{u'\left[\frac{\varphi_x^T h_x^T}{\tau}\right] \frac{\varphi_x^T h_x^T}{\tau} - \frac{Q^{\mathrm{MK}}(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau)}{L} h_x^T} = \frac{f_d}{f_x}.$$

The productivity levels and employment levels of the least productive domestic firm and exporter must also be such that the labor market clearing condition is satisfied:

$$L = \left[\begin{array}{l} M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_d^T}^{\varphi_{\max}} h_d\left(\varphi, \varphi_d^T, h_d^T\right) \mathrm{d}G(\varphi) + M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \int_{\varphi_x^T}^{\varphi_{\max}} h_x\left(\varphi, \varphi_x^T, h_x^T, \tau\right) \mathrm{d}G(\varphi) \\ \\ +M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \left[1 - G\left(\varphi_d^T\right)\right] f_d + M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) \left[1 - G\left(\varphi_x^T\right)\right] f_x \\ \\ +M_e(\varphi_d^T, h_d^T, \varphi_x^T, h_x^T, \tau) f_e \end{array} \right].$$

References

[1] Anderson, James E. and Eric van Wincoop (2004) "Trade Costs," *Journal of Economic Literature*, 42(3), 691-751.

[2] Arkolakis, Costas, Arnaud Costinot and Andrés Rodriguez-Clare (2012) "New Trade Models, Same Old Gains," *American Economic Review*, 102(1), 94-130.

[3] Atkeson Andrew and Ariel Burstein (2010) "Innovation, Firm Dynamics and International Trade," *Journal of Political Economy*, 118(3), 433-484.

[4] Axtell, R. L. (2001) "Zipf Distribution of US Firm Sizes," *Science*, 293, 1818-20.

[5] Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel S. Kortum (2003) "Plants and Productivity in International Trade," *American Economic Review*, 93(4), 1268-1290.

[6] Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding and Peter K. Schott (2007) "Firms in International Trade," *Journal of Economic Perspectives*, 21(3), 105-130.

[7] Costinot, Arnaud and Andres Rodriguez-Clare (2014) "Trade Theory with Numbers: Quantifying the Consequences of Globalization," *Handbook of International Economics*, Chapter 4, 197-262.

[8] Das, Mita, Mark J. Roberts and James R. Tybout (2007) "Market Entry Costs, Producer Heterogeneity and Export Dynamics," *Econometrica*, 75(3), 837-73.

[9] Dhingra, Swati and John Morrow (2012) "Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity," London School of Economics, mimeograph.

[10] Dixit, Avinash K. and Joseph E. Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297-308.

[11] Eaton, Jonathan and Kortum, Samuel (2002) "Technology, Geography, and Trade," *Econometrica*, 70(5), 1741-1779.

[12] Feenstra, Robert C. (2010) "Measuring the Gains from Trade under Monopolistic Competition," *Canadian Journal of Economics*, February, 1-28.

[13] Feenstra, Robert C. (2014) "Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity," *NBER Working Paper*, 19833.

[14] Head, Keith, Thierry Mayer and Mathias Thoenig (2014) "Welfare and Trade Without Pareto," *American Economic Review*, Papers and Proceedings, 104(5), 310-16.

[15] Helpman Elhanan, Melitz Marc J. and Yona Rubinstein (2008) "Estimating Trade Flows: Trading Partners and Trading Volumes," *Quarterly Journal of Economics*, 123(2), 441-87.

[16] Hsieh, Chang-Tai and Ralph Ossa (2011) "A Global View of Productivity Growth in China," *NBER Working Paper*, 16778.

[17] Imbs, Jean and Isabelle Méjean (2009) "Elasticity Optimism," *CEPR Discussion Paper*, 7177.

[18] Krugman, Paul. R. (1980) "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70(5), 950-959.

[19] Melitz, Marc J. (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695-1725.

[20] Melitz, Marc J. and Stephen J. Redding (2014) “Heterogeneous Firms and Trade,” *Handbook of International Economics*, Volume 4, Chapter 1, 1-54.

[21] Novy, Dennis (2013) “International Trade without CES: Estimating Translog Gravity,” *Journal of International Economics*, 89(2), 271-282.

[22] Ossa, Ralph (2012) “Why Trade Matters After All,” *NBER Working Paper*, 18113.

[23] Roberts, Mark J. and James R. Tybout (1997) “The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs,” *American Economic Review*, 87(4), 545-564.

[24] Simonovska, Ina and Michael E. Waugh (2014a) “The Elasticity of Trade: Estimates and Evidence,” *Journal of International Economics*, 92(1), 34-50.

[25] Simonovska, Ina and Michael E. Waugh (2014b) “Trade Models, Trade Elasticities, and the Gains from Trade,” University of California at Davis, mimeograph.

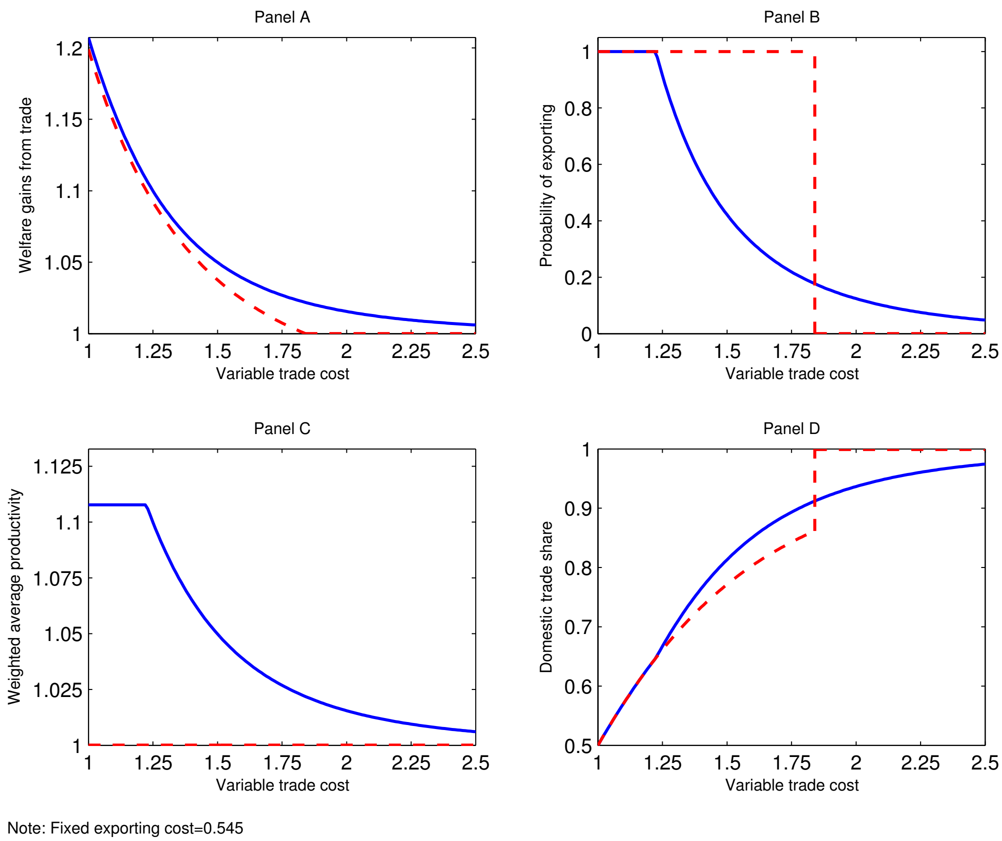

Figure A.1: Reductions in variable trade costs from the closed economy equilibrium

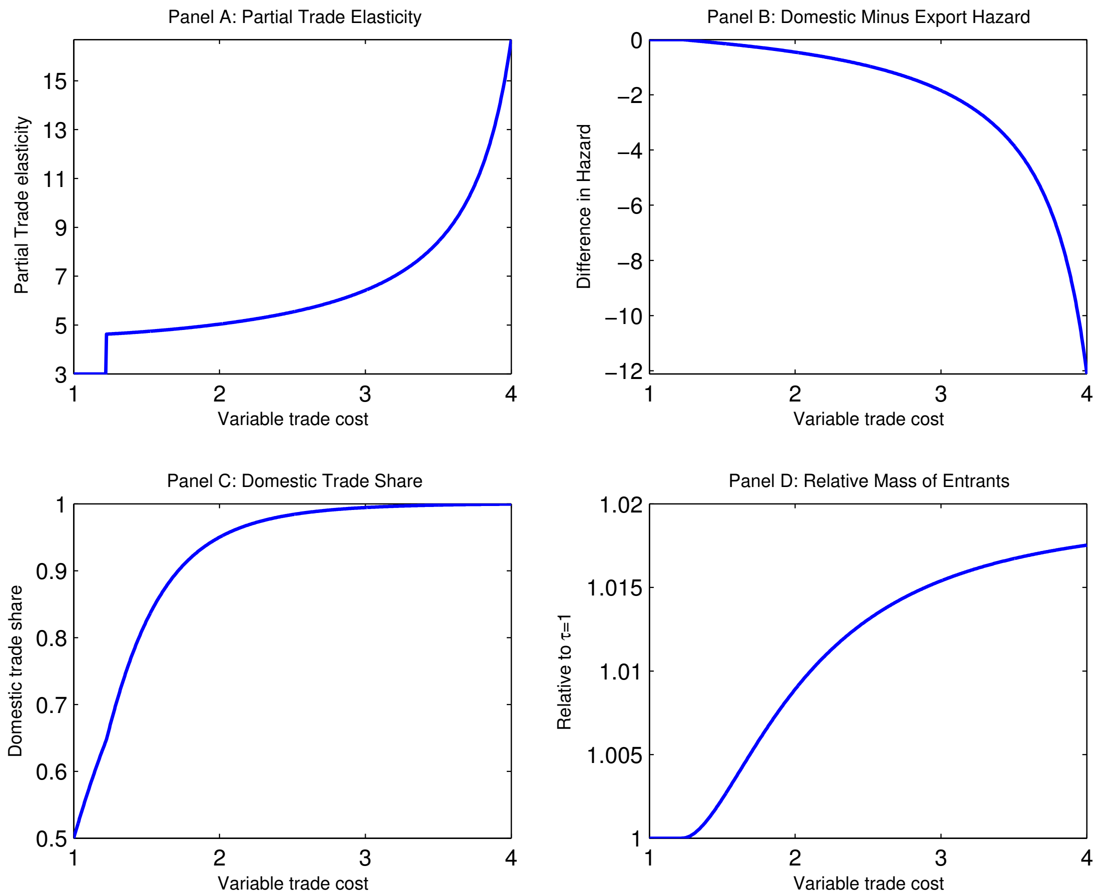

Figure A.2: Trade elasticity, domestic trade share and mass of entrants in the heterogeneous firm model with a truncated Pareto productivity distribution